Intrinsic Fisher-Kähler Information Geometry

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Abstract

We construct a Fisher-Kähler information geometry on coadjoint orbits of the unitary group and show how it controls both reversible and irreversible quantum dynamics. Starting from the Bogoliubov-Kubo-Mori information metric on density operators and the Kirillov-Kostant-Souriau symplectic form on fixed-spectrum orbits, we define the Fisher structure $B = g^{-1}\omega$, show that $-B^2$ is negative definite with an explicit spectral decomposition, and obtain a canonical untwisting to a Kähler triple (h, ω, I). We derive formulas for Fisher-Kähler gradients and Hamiltonian vector fields, with particular emphasis on the linear energy functional $E(\rho) = \mathrm{Tr}(H\rho)$ and on the unified generator $K = G + J$ that realises hypocoercive, metriplectic dynamics of a single information current seen in two quadratures. We phrase this as a universal principle for Universal Information Hydrodynamics (UIH): a wide class of reversible and irreversible evolutions in physics are generated by a single information current on a Fisher-Kähler state space. Within this framework we reinterpret Frieden's Extreme Physical Information (EPI): the functionals I and J_F are two quadratures of the same current. For one-dimensional translation families we prove an exact EPI-to-UIH Fisher identity equating parametric and spatial Fisher informations, $I_{\mathrm{param}} = I_x$, and verify it numerically for Gaussian, Laplace, and Cauchy laws. The symmetric part G of K defines Fisher spectral channels whose rate-density tails control early-time information reception; power-law tails generate universal growth exponents, including a golden exponent channel from a simple two-channel renormalisation map. To anchor the construction in experiment we build Fisher channels from two data sets: strange metal magnetotransport, where we treat $\rho_{zz}(\theta)$ as a circular density to extract angular Fisher information I_θ and its spectral fingerprints, and optically trapped microspheres, where an Ornstein-Uhlenbeck fit to centre-of-mass trajectories confirms that the analytic Fisher information $1/\sigma^2$ agrees with both parametric and grid-based estimates. As a worked EPI-style sector we outline a bounded Fisher entropy functional for a scalar vacuum field whose free energy admits a Bogomolny-type completion; in the zero-temperature limit this becomes a pure Fisher BPS functional whose minimisers are Fisher halos, providing a rigid template for later applications and a geometric realisation of Fisher-based relaxation.

Contents

1 Universal geometric principle

Universal Information Hydrodynamics is the statement that a wide class of reversible and irreversible evolutions in physics are generated by a single underlying information current, seen in two quadratures, on a Fisher-Kähler state space.

The aim of this section is to collect the geometric ingredients, the dynamical form, and the variational and spectral consequences into a single unified principle. The concrete coadjoint orbit geometry will be developed in Section 2 and can be viewed as the main finite dimensional realisation of the abstract structures introduced here.

1.1 State space and Fisher-Kähler structure

Let M denote a state space of interest.

- In the classical setting, M is a suitable space of probability densities ρ on a configuration manifold X.
- In the quantum setting, M is a coadjoint orbit of density matrices of fixed spectrum acting on a Hilbert space $\mathcal{H}$. We will primarily consider the finite dimensional case, where $\mathcal{H} \cong \mathbb{C}^d$ and the orbit lives inside the manifold of faithful density operators $\mathcal{D}^{\times}$.

In both cases we assume that M carries the following structures.

(i) A monotone information metric g. Classically this is the Fisher information metric on densities [5]. Quantum mechanically it is the Bogoliubov-Kubo-Mori (BKM) metric on faithful density operators [8]. The metric g measures statistical distinguishability of nearby states and is monotone under coarse graining and completely positive trace preserving (CPTP) maps.

(ii) A symplectic form ω on M capturing the reversible, Hamiltonian aspect of the dynamics. Classically this is the usual symplectic form on phase space or its pushforward to density space. Quantum mechanically it is the KKS form on coadjoint orbits of the unitary group.

(iii) A compatibility condition tying g and ω together via the *information structure tensor*

$$B_\rho \;:=\; g_\rho^{-1}\,\omega_\rho \;:\; T_\rho M \longrightarrow T_\rho M,$$

defined pointwise for $\rho \in M$. Here $g_\rho^{-1} : T_\rho^* M \to T_\rho M$ is the musical isomorphism associated to the metric, and ω_ρ is viewed as a map $T_\rho M \to T_\rho^* M$ via $X \mapsto \omega_\rho(X, \cdot)$.

We make the following structural assumption.

Definition 1.1 (*Fisher structure*). *A* Fisher structure *on* (M, g, ω) *is an endomorphism* B *of the tangent bundle* TM *such that*

$$B_\rho \;=\; g_\rho^{-1}\,\omega_\rho,$$

and for each $\rho \in M$ *the operator* $-B_\rho^2$ *is positive definite on* $T_\rho M$.

The positivity of $-B_\rho^2$ means that B_ρ has purely imaginary eigenvalues and no real kernel. In the finite dimensional quantum orbit case this will be shown explicitly in Section 2 by diagonalising B_ρ on root planes. At the abstract level this allows us to define a positive intertwiner

$$S_\rho \;\coloneqq\; \sqrt{-B_\rho^2},$$

and from this a twisted complex structure

$$I_\rho \;\coloneqq\; S_\rho^{-1}\, B_\rho.$$

By construction $I_\rho^2 = -\mathbf{1}_{T_\rho M}$, so I defines an almost complex structure on M. Using S_ρ we can also define a new Riemannian metric

$$h_\rho(X,Y) \;\coloneqq\; g_\rho(S_\rho X, Y), \qquad X, Y \in T_\rho M.$$

Definition 1.2 (*Fisher-Kähler structure*). *A* Fisher-Kähler structure *on M is a triple (h, ω, I) obtained from a Fisher structure B as above such that*

- *(a) (M, h, I) is a Kähler manifold, that is I is integrable, $h(IX, IY) = h(X, Y)$, and the fundamental two form $\Omega(X,Y) \coloneqq h(X, IY)$ is closed;*
- *(b) $\omega = \Omega$, so the original symplectic form coincides with the Kähler form derived from h and I.*

In this situation we say that the pair (g, ω) admits a canonical Fisher-Kähler untwisting and that B encodes the two quadratures of a single underlying information current, seen through the symmetric metric h and the antisymmetric form ω.

At this level of generality we will not attempt to prove integrability of I for all possible choices of (M, g, ω). Instead we will show in Section 2 that for finite dimensional quantum state orbits the construction above does produce a genuine Kähler structure (h, ω, I), and we will regard this as the main motivating example for the universal principle.

1.2 Gradient and Hamiltonian flows

Once a Fisher-Kähler structure (h, ω, I) is given, any sufficiently regular functional $\Phi \colon M \to \mathbb{R}$ generates two canonical vector fields:

- the *Fisher-Kähler gradient*

$$\nabla_h \Phi(\rho) \;\in\; T_\rho M, \qquad h_\rho\big(\nabla_h \Phi(\rho), X\big) \;=\; \mathrm{d}\Phi_\rho(X) \quad \forall\, X \in T_\rho M,$$

  which generates dissipative, entropy increasing flow;
- the *Hamiltonian vector field*

$$X_\Phi(\rho) \;\in\; T_\rho M, \qquad \omega_\rho\big(X_\Phi(\rho), X\big) \;=\; \mathrm{d}\Phi_\rho(X) \quad \forall\, X \in T_\rho M,$$

  which generates reversible, symplectic evolution.

Equivalently, at the operator level one can encode these in a unified generator K acting on functionals $F\colon M \to \mathbb{R}$ via

$$\frac{\mathrm{d}}{\mathrm{d}t}F(\rho_t) \;=\; \{F,\Phi\}_\omega(\rho_t) \;-\; \big\langle \nabla_h F(\rho_t),\, \nabla_h \Phi(\rho_t)\big\rangle_{h_{\rho_t}},$$

where $\{\,\cdot\,,\,\cdot\,\}_\omega$ is the Poisson bracket induced by ω. In terms of a linear operator K acting on (co)vectors one can write, very schematically,

$$K \;=\; G + J, \qquad G = G^\top \le 0, \qquad J = -J^\top,$$

with the symmetric part G determined by the Fisher-Kähler metric and the antisymmetric part J determined by the symplectic form. The induced flow on states can be written as

$$\dot\rho_t \;=\; -\nabla_h\Phi(\rho_t) \;+\; X_\Phi(\rho_t),$$

with entropy production governed by the Dirichlet form associated to G and reversible motion governed by J.

In concrete quantum and classical settings the operator K can be realised as a Fokker-Planck, Lindblad, or more general Markov generator on the underlying state space, and the Fisher-Kähler metric h appears as the metric that makes these generators hypocoercive and metriplectic [16]. We will see in Section 2 how this works explicitly on finite dimensional quantum orbits.

1.3 Variational and spectral consequences

The unified form (1.2) and the Fisher-Kähler geometry admit two complementary viewpoints that will be important later.

Variational viewpoint. Given a functional Φ on M we can consider the pure gradient flow

$$\dot\rho_t \;=\; -\nabla_h\Phi(\rho_t).$$

Stationary points satisfy $\nabla_h\Phi(\rho_*) = 0$ and are critical points of Φ. Under suitable convexity conditions on Φ the flow converges to minimisers, and the Fisher-Kähler structure provides a canonical metric for this optimisation.

In Frieden's Extreme Physical Information (EPI) framework one introduces two functionals $I[\rho]$ and $J_F[\rho]$, interpreted as data information and source information, and obtains physical field equations by extremising

$$K_{\mathrm{EPI}}[\rho] \;=\; I[\rho] - J_F[\rho]$$

under appropriate constraints, $\delta(I - J_F) = 0$. In the UIH setting this EPI principle can be reinterpreted geometrically as a special choice of potential Φ. To avoid a clash with the antisymmetric part J of the unified generator $K = G + J$, we reserve the notation J_F for Frieden's source information functional throughout.

The Fisher functional I generates a pure gradient flow via $-\nabla_h I$, and its second variation at a stationary point ρ_* defines the Dirichlet form associated with the symmetric part

G,

$$\delta^2 I[\rho_*](\delta\rho, \delta\rho) \;=\; \langle \delta\rho,\, G\,\delta\rho \rangle_{h_{\rho_*}},$$

so that I is precisely the Fisher quadratic form of the dissipative channel. Structural functionals such as J_F often admit a Hamiltonian representation via X_{J_F}. Choosing

$$\Phi \;=\; I - J_F$$

in (1.2) leads to

$$\dot{\rho}_t \;=\; -\nabla_h I(\rho_t) \;+\; \nabla_h J_F(\rho_t) \;+\; X_{I-J_F}(\rho_t),$$

and stationary points of the flow coincide with solutions of the EPI variational problem. The Fisher-Kähler geometry thus supplies the metric and symplectic structure behind the EPI functionals.

Spectral viewpoint. The symmetric part G of the generator K defines a nonpositive selfadjoint operator on an appropriate Hilbert space of fluctuations around equilibrium. Its spectrum encodes a distribution of relaxation rates, and the Fisher-Kähler metric h controls the associated Dirichlet form. If we isolate a particular slow channel, for example by projecting onto a subspace spanned by some observable or mode family, the restriction of G defines an effective rate density $\rho_{\text{eff}}(\lambda)$ for that channel.

Given a nonnegative test function f of time we consider quantities of the form

$$I(t) \;=\; \int_0^\infty \left(1 - e^{-\lambda t}\right) \rho_{\text{eff}}(\lambda)\, \mathrm{d}\lambda,$$

which model information reception over time in the slow channel. If the rate density has a power law tail

$$\rho_{\text{eff}}(\lambda) \;\sim\; C\,\lambda^{-1-\delta} \qquad \text{for large } \lambda,$$

then Tauberian arguments imply an early time asymptotic

$$I(t) \;\propto\; t^{\delta} \qquad \text{as } t \downarrow 0.$$

In this way the Fisher spectral tail of a slow sector is directly reflected in a growth exponent for information reception, and particular exponents (such as the golden ratio case) correspond to specific spectral fingerprints of G. We will return to this spectral story in more detail in Section 5.

1.4 Preview of the scalar Fisher sector

In later work on UIH gravity the abstract structures above are instantiated in a scalar Fisher sector modelling a coarse grained "vacuum" degree of freedom. The relevant state variable is a scalar field $\sigma(x)$ whose gradient energy is measured by a Fisher functional and whose local occupation statistics are encoded by a bounded entropy functional. The associated free energy can be written in a Bogomolny type form

$$F[\sigma] \;=\; \frac{1}{2}\,\|\nabla\sigma - q\|_w^2 \;-\; T_F\, S_{\text{bnd}}[\sigma],$$

where q is a baryonic source, w is a Fisher weight, and S_{bnd} is a bounded entropy built from a sigmoid map of σ. In the zero temperature limit $T_F \to 0$ one recovers a pure Fisher BPS functional whose minimisers are Fisher halos; for $T_F > 0$ the bounded entropy deforms and truncates these halos.

From the present point of view this scalar sector is simply a particularly rigid example of an EPI style functional on a Fisher-Kähler state space, with the Fisher part and the source part geometrically anchored by the structures above. The details of this construction will be developed later in Section 6.

2 Fisher-Kähler geometry on quantum state orbits

We now turn to the main finite dimensional realisation of the abstract structures introduced above: the manifold of quantum states of fixed spectrum, viewed as a coadjoint orbit of the unitary group, equipped with the Bogoliubov-Kubo-Mori metric and the KKS symplectic form. On this manifold the Fisher structure $B = g^{-1}\omega$ can be diagonalised explicitly, and the Fisher-Kähler untwisting $(g, \omega, B) \mapsto (h, \omega, I)$ can be carried out in closed form.

2.1 Quantum state space and coadjoint orbits

Let $\mathcal{H}$ be a complex Hilbert space of dimension d and let

$$\mathcal{D}^\times \;:=\; \big\{\rho \in \mathcal{B}(\mathcal{H}) \,\big|\, \rho^\dagger = \rho,\; \rho > 0,\; \operatorname{Tr}\rho = 1\big\}$$

denote the manifold of faithful density operators on $\mathcal{H}$. This is an open subset of the affine hyperplane of Hermitian trace one operators in the space of all bounded operators on $\mathcal{H}$ and carries a natural smooth manifold structure.

Fix a list of eigenvalues

$$\lambda \;=\; (\lambda_1, \dots, \lambda_d), \qquad \lambda_i > 0, \qquad \sum_{i=1}^{d} \lambda_i = 1,$$

and assume for simplicity that the spectrum is nondegenerate, so $\lambda_i \neq \lambda_j$ for $i \neq j$. Let

$$\rho_\lambda \;:=\; \operatorname{diag}(\lambda_1, \dots, \lambda_d)$$

in some fixed orthonormal basis of $\mathcal{H}$. The unitary group $U(\mathcal{H})$ acts on $\mathcal{D}^\times$ by conjugation,

$$U \cdot \rho \;:=\; U\rho U^\dagger, \qquad U \in U(\mathcal{H}),$$

and the orbit of ρ_λ under this action is

$$\mathcal{O}_\lambda \;:=\; \{U\rho_\lambda U^\dagger \,|\, U \in U(\mathcal{H})\} \;\subset\; \mathcal{D}^\times.$$

This orbit consists of all density operators on $\mathcal{H}$ with spectrum equal to λ, and is a smooth compact homogeneous manifold of real dimension $d^2 - d$.

The tangent space at a point $\rho \in \mathcal{O}_\lambda$ can be identified with commutators.

Proposition 2.1 . *For $\rho \in \mathcal{O}_\lambda$ the tangent space is*

$$T_\rho\mathcal{O}_\lambda \;=\; \{i[H_0,\rho] \,|\, H_0^\dagger = H_0\}.$$

Proof. The orbit map $U \mapsto U\rho U^\dagger$ has derivative at the identity given by

$$\frac{\mathrm{d}}{\mathrm{d}t}\Big|_{t=0} e^{itH_0}\,\rho\,e^{-itH_0} \;=\; i[H_0,\rho]$$

for any Hermitian H_0. This spans the tangent space at ρ. Surjectivity follows from the general theory of homogeneous spaces [11] or by dimension counting. □

In what follows we will frequently work in the eigenbasis of ρ. Writing

$$\rho \;=\; U\rho_\lambda U^\dagger \;=\; U\,\mathrm{diag}(\lambda_1,\ldots,\lambda_d)\,U^\dagger,$$

we denote by

$$\tilde{X} \;:=\; U^\dagger X U$$

the matrix of an operator X in this eigenbasis and write $\tilde{X}_{ij}$ for its entries. On the orbit $\mathcal{O}_\lambda$ the tangent vectors $X \in T_\rho\mathcal{O}_\lambda$ are represented in the eigenbasis of ρ by Hermitian matrices with vanishing diagonal:

$$\tilde{X}_{ii} = 0, \qquad \tilde{X}_{ij} = \overline{\tilde{X}_{ji}} \quad (i \neq j).$$

2.2 The BKM information metric

The Bogoliubov-Kubo-Mori information metric on $\mathcal{D}^\times$ can be defined in several equivalent ways. For our purposes a convenient expression is

$$g_\rho(X,Y) \;=\; \int_0^1 \mathrm{Tr}\big(\rho^t X \rho^{1-t} Y\big)\,\mathrm{d}t, \qquad X,Y \in T_\rho\mathcal{D}^\times,$$

which is known to be the unique monotone Riemannian metric on $\mathcal{D}^\times$ [7] that yields the quantum relative entropy as a Bregman divergence in the affine structure of density operators. Restricted to the orbit $\mathcal{O}_\lambda$ it is strictly positive definite on $T_\rho\mathcal{O}_\lambda$.

Working in the eigenbasis of ρ we can express g_ρ in terms of the entries of $\tilde{X}$ and $\tilde{Y}$. One finds

$$g_\rho(X,Y) \;=\; \sum_{i<j} c_{ij}\big(\tilde{X}_{ij}\,\overline{\tilde{Y}_{ij}} + \overline{\tilde{X}_{ij}}\,\tilde{Y}_{ij}\big),$$

where the coefficients

$$c_{ij} \;=\; c(\lambda_i, \lambda_j) \;:=\; \frac{\log \lambda_i - \log \lambda_j}{\lambda_i - \lambda_j}$$

are the logarithmic means of the eigenvalues. In particular $c_{ij} > 0$ and $c_{ij} = c_{ji}$. It is natural to organise the summation over *root planes*, two dimensional real subspaces of $T_\rho O_\lambda$ associated with each unordered pair (i, j), as we will do below when we introduce the KKS form and the Fisher structure.

The expression (2.2) makes it clear that g_ρ is block diagonal in the decomposition of $T_\rho O_\lambda$ into real two dimensional subspaces associated with each unordered pair (i, j). This root plane decomposition will be used heavily in what follows.

2.3 The KKS symplectic form

The coadjoint orbit O_λ carries a natural symplectic structure, the Kirillov-Kostant-Souriau (KKS) form. At a point $\rho \in O_\lambda$ it is defined on tangent vectors of the form $X = i[A, \rho]$ and $Y = i[B, \rho]$ by

$$\omega_\rho(X, Y) \;:=\; i \,\mathrm{Tr}\big(\rho [A, B]\big) \;=\; i \,\mathrm{Tr}\big([\rho, A]\, B\big),$$

with A and B Hermitian. This form is well defined on $T_\rho O_\lambda$, skew symmetric, nondegenerate, and closed, making (O_λ, ω) into a compact symplectic manifold.

To express ω_ρ in the eigenbasis of ρ we again write ρ in diagonal form and denote by $\tilde{X}$ and $\tilde{Y}$ the matrices of X and Y in this basis. A short computation shows that

$$\omega_\rho(X, Y) \;=\; 2 \sum_{i<j} (\lambda_i - \lambda_j)\, \Im\big(\tilde{X}_{ij}\, \overline{\tilde{Y}_{ij}}\big),$$

where $\Im(z)$ denotes the imaginary part of a complex number z. Thus the KKS form is also block diagonal in the root plane decomposition of $T_\rho O_\lambda$.

2.4 Root plane decomposition

The expressions (2.2) and (2.3) show that both g_ρ and ω_ρ decompose into a direct sum of two dimensional real blocks labelled by unordered pairs (i, j) of distinct indices. It is convenient to fix a real basis on each such block.

For each pair (i, j) with $i < j$ define real tangent vectors E_{ij} and F_{ij} at ρ via their matrices in the eigenbasis of ρ:

$$\tilde{E}_{ij} \;:=\; \begin{pmatrix} 0 & \cdots & 0 & 1 & 0 & \cdots & 0 \\ \vdots & & & & & & \vdots \\ 0 & \cdots & 0 & 0 & 0 & \cdots & 0 \\ 1 & \cdots & 0 & 0 & 0 & \cdots & 0 \\ 0 & \cdots & 0 & 0 & 0 & \cdots & 0 \\ \vdots & & & & & & \vdots \end{pmatrix}, \qquad \tilde{F}_{ij} \;:=\; \begin{pmatrix} 0 & \cdots & 0 & i & 0 & \cdots & 0 \\ \vdots & & & & & & \vdots \\ 0 & \cdots & 0 & 0 & 0 & \cdots & 0 \\ -i & \cdots & 0 & 0 & 0 & \cdots & 0 \\ 0 & \cdots & 0 & 0 & 0 & \cdots & 0 \\ \vdots & & & & & & \vdots \end{pmatrix},$$

with all entries zero except at the (i, j) and (j, i) positions as shown. In terms of these basis vectors the restrictions of g_ρ and ω_ρ to the root plane spanned by $\{E_{ij}, F_{ij}\}$ take the form

$$g_\rho\big|_{ij} \;=\; 2c_{ij}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \qquad \omega_\rho\big|_{ij} \;=\; 2(\lambda_i - \lambda_j)\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix},$$

where c_{ij} is the logarithmic mean defined above. This makes the subsequent analysis of the Fisher structure $B_\rho = g_\rho^{-1}\omega_\rho$ essentially algebraic.

2.5 The Fisher structure on the orbit

On the tangent space $T_\rho \mathcal{O}_\lambda$ we define the Fisher structure as in Definition 1.1,

$$B_\rho \;:=\; g_\rho^{-1}\,\omega_\rho.$$

Using the root plane decomposition and the block forms (2.4) we can compute B_ρ explicitly on each block:

$$B_\rho\big|_{ij} \;=\; g_\rho^{-1}\big|_{ij}\,\omega_\rho\big|_{ij} \;=\; \frac{1}{2c_{ij}}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \cdot 2(\lambda_i - \lambda_j)\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \;=\; \beta_{ij}\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix},$$

where

$$\beta_{ij} \;:=\; \frac{\lambda_i - \lambda_j}{c_{ij}} \;=\; \frac{(\lambda_i - \lambda_j)^2}{\log\lambda_i - \log\lambda_j}$$

are the *Fisher weights*. The matrix

$$J_0 \;:=\; \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

is the standard complex structure on $\mathbb{R}^2$, satisfying $J_0^2 = -\mathbf{1}$. Thus on each root plane the Fisher structure has the simple form

$$B_\rho\big|_{ij} \;=\; \beta_{ij} J_0.$$

It follows immediately that

$$-B_\rho^2\big|_{ij} \;=\; \beta_{ij}^2\mathbf{1},$$

so $-B_\rho^2$ is positive definite on each root plane and hence on all of $T_\rho O_\lambda$.

Proposition 2.2 . *On the coadjoint orbit O_λ equipped with the BKM metric g and the KKS form ω, the information structure tensor $B_\rho = g_\rho^{-1}\,\omega_\rho$ defines a Fisher structure in the sense of Definition 1.1. In particular, $-B_\rho^2$ is positive definite on $T_\rho O_\lambda$ for every $\rho \in O_\lambda$.*

Proof. The block form (2.5) shows that on each root plane $-B_\rho^2\big|_{ij} = \beta_{ij}^2\mathbf{1}$ with $\beta_{ij}^2 > 0$ for $\lambda_i \neq \lambda_j$. Since tangent vectors on O_λ have vanishing diagonal components in the eigenbasis, $T_\rho O_\lambda$ is the direct sum of these two dimensional blocks and $-B_\rho^2$ is positive definite on each block. This proves the claim. □

2.6 Fisher untwisting and the Kähler triple

Given the Fisher structure B_ρ and the positivity of $-B_\rho^2$ we can define the positive intertwiner

$$S_\rho \;:=\; \sqrt{-B_\rho^2}$$

uniquely as a positive definite operator on $T_\rho O_\lambda$. On each root plane we have

$$S_\rho\big|_{ij} \;=\; \sqrt{-B_\rho^2\big|_{ij}} \;=\; |\beta_{ij}|\,\mathbf{1}.$$

In terms of S_ρ we define the untwisted complex structure

$$I_\rho \;:=\; S_\rho^{-1}B_\rho$$

and the Fisher-Kähler metric

$$h_\rho(X,Y) \;:=\; g_\rho(S_\rho X, Y).$$

On each root plane this gives

$$I_\rho\big|_{ij} \;=\; \mathrm{sgn}(\beta_{ij})\,J_0,$$

and

$$h_\rho\big|_{ij} \;=\; 2c_{ij}|\beta_{ij}|\mathbf{1} \;=\; 2c_{ij}\left|\frac{\lambda_i-\lambda_j}{c_{ij}}\right|\mathbf{1} \;=\; 2|\lambda_i-\lambda_j|\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}.$$

Remarkably, the logarithmic mean c_{ij} cancels out, leaving a Fisher-Kähler metric determined solely by the linear spectral gaps. In particular $I_\rho^2 = -\mathbf{1}$ and h_ρ is positive definite.

Theorem 2.3 (*Fisher-Kähler structure on the orbit*). *For the coadjoint orbit O_λ of faithful density operators of fixed nondegenerate spectrum, equipped with the BKM metric g and the KKS symplectic form ω, the triple (h,ω,I) defined from the Fisher structure $B = g^{-1}\omega$ is a Kähler structure in the sense of Definition 1.2. Moreover, the Kähler form associated to (h,I) coincides with the original symplectic form ω.*

Proof. The fact that $I_\rho^2 = -\mathbf{1}$ and that h_ρ is positive definite follows from the block analysis above. Compatibility of h and I, in the sense that $h_\rho(I_\rho X, I_\rho Y) = h_\rho(X, Y)$, holds blockwise because I_ρ acts as J_0 or $-J_0$ and J_0 is orthogonal with respect to the Euclidean metric.

To identify the Kähler form we compute, for $X, Y \in T_\rho O_\lambda$,

$$\Omega_\rho(X,Y) \;:=\; h_\rho(X, I_\rho Y) \;=\; g_\rho(S_\rho X, I_\rho Y) \;=\; g_\rho(B_\rho X, Y) \;=\; \omega_\rho(X,Y),$$

where we used $I_\rho = S_\rho^{-1} B_\rho$ and $B_\rho = g_\rho^{-1}\omega_\rho$. Thus $\Omega = \omega$ and in particular Ω is closed because ω is.

Integrability of I can be established using standard results on homogeneous Kähler manifolds: coadjoint orbits of compact semisimple Lie groups admit a unique invariant complex structure compatible with the KKS symplectic form, and the Fisher untwisting construction recovers this complex structure. We refer to the literature on coadjoint orbits and geometric quantisation for details [12]. This proves that $(O_\lambda, h, \omega, I)$ is Kähler. □

The theorem shows that the pair (g, ω) on the quantum state orbit admits a canonical Fisher-Kähler untwisting to (h, ω, I), with B encoding the two quadratures of the information current. The metric h will serve as the natural information metric for gradient flows, while ω and I continue to encode the Hamiltonian structure.

2.7 Fisher-Kähler gradients and Hamiltonian fields

Let $\Phi\colon O_\lambda \to \mathbb{R}$ be a smooth functional. The Fisher-Kähler gradient $\nabla_h \Phi(\rho)$ is defined by

$$h_\rho\big(\nabla_h\Phi(\rho), X\big) \;=\; \mathrm{d}\Phi_\rho(X) \qquad \forall\, X \in T_\rho O_\lambda,$$

and the Hamiltonian vector field $X_\Phi(\rho)$ by

$$\omega_\rho\big(X_\Phi(\rho), X\big) \;=\; \mathrm{d}\Phi_\rho(X) \qquad \forall\, X \in T_\rho O_\lambda.$$

Using the Fisher structure B_ρ and the intertwiner S_ρ we can express these vector fields more directly in terms of g_ρ.

First, note that

$$\omega_\rho(X,Y) \;=\; g_\rho(B_\rho X, Y) \;=\; g_\rho(S_\rho I_\rho X, Y) \;=\; h_\rho(I_\rho X, Y),$$

so that

$$\omega_\rho\big(X_\Phi(\rho), X\big) \;=\; h_\rho\big(I_\rho X_\Phi(\rho), X\big) \;=\; \mathrm{d}\Phi_\rho(X).$$

By nondegeneracy of h_ρ this implies

$$I_\rho X_\Phi(\rho) \;=\; \nabla_h\Phi(\rho), \qquad X_\Phi(\rho) \;=\; -I_\rho \nabla_h \Phi(\rho).$$

Thus on a Fisher-Kähler manifold the Hamiltonian vector field associated with Φ is obtained from the Fisher-Kähler gradient by rotation with the complex structure.

On the orbit, and in particular for functionals of the form

$$E(\rho) \;:=\; \mathrm{Tr}(H\rho),$$

one can write $\nabla_h E(\rho)$ and $X_E(\rho)$ in terms of commutators with ρ and a suitable effective Hamiltonian that depends on both H and the spectrum of ρ. We will return to these explicit formulas in Section 2, after discussing the classical limit.

2.8 Classical limit and the probability simplex

Before turning to dynamics it is useful to recall how the quantum construction above reduces to the classical Fisher geometry on the probability simplex.

Let

$$\Delta^{n-1} \;:=\; \Big\{p = (p_1, \dots, p_n) \in (0,1)^n \,\Big|\, \sum_{i=1}^{n} p_i = 1\Big\}$$

denote the open probability simplex. A tangent vector at p is a vector $v \in \mathbb{R}^n$ with zero sum, $\sum_i v_i = 0$. The Fisher information metric on Δ^{n-1} is

$$g_p(v, w) \;=\; \sum_{i=1}^{n} \frac{v_i w_i}{p_i},$$

which is the classical analogue of the BKM metric. There is no intrinsic symplectic form on Δ^{n-1}, but when the simplex is embedded into a Hamiltonian phase space the restriction of the ambient symplectic structure induces a two form on the image, which can be pushed forward to Δ^{n-1}.

In the commutative case the coadjoint orbit construction collapses: density matrices are diagonal in a fixed basis, commutators vanish, and the KKS form is trivial. The Fisher structure $B = g^{-1}\omega$ therefore vanishes, and the untwisting procedure yields $S = \mathbf{1}$, $I = 0$, and $h = g$. Thus the Fisher-Kähler structure degenerates to the pure Fisher metric on Δ^{n-1}, as expected for a purely classical statistical manifold without intrinsic Hamiltonian structure.

This degeneration illustrates a general pattern. The noncommutative degrees of freedom in the quantum case are responsible for the nontrivial KKS form, the Fisher structure, and the complex structure I. When these are absent the geometry simplifies to the classical Fisher metric, and the Fisher-Kähler picture reduces to ordinary information geometry. In the remainder of the paper we will keep both regimes in mind, with the coadjoint orbit geometry providing the canonical finite dimensional Fisher-Kähler benchmark.

2.9 Explicit gradients for linear observables

We now specialise the general Fisher-Kähler gradient and Hamiltonian vector field to the linear energy functional

$$E(\rho) \;:=\; \mathrm{Tr}(H\rho),$$

with H a fixed Hermitian operator on $\mathcal{H}$. This case is particularly important, as it underlies both reversible Schrödinger evolution and irreversible relaxation driven by an energy functional.

Let $\rho \in \mathcal{O}_\lambda$ with eigenbasis U and eigenvalues λ_i. We denote $\tilde{H} = U^\dagger H U$ and write $\tilde{H}_{ij}$ for its matrix elements. Tangent vectors are represented in this basis by Hermitian matrices with zero diagonal, as before.

A standard calculation shows that the differential of E at ρ applied to a tangent vector $X \in T_\rho \mathcal{O}_\lambda$ is

$$\mathrm{d}E_\rho(X) \;=\; \mathrm{Tr}(HX) \;=\; 2\sum_{i<j} \Re\big(\tilde{H}_{ij}\,\overline{\tilde{X}_{ij}}\big),$$

where $\Re(z)$ denotes the real part. Comparing this with the simplified block form of the Fisher-Kähler metric,

$$h_\rho\big|_{ij} \;=\; 2|\lambda_i - \lambda_j| \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix},$$

we see that on each root plane the gradient $\nabla_h E(\rho)$ has components proportional to the off diagonal entries of H,

$$(\nabla_h E(\rho))_{ij} \;=\; \frac{1}{|\lambda_i - \lambda_j|}\,\tilde{H}_{ij}, \qquad i \neq j.$$

This confirms that the dissipative flow is governed strictly by the energy differences of the eigenstates. we see that on each root plane the gradient $\nabla_h E(\rho)$ has components proportional to the off diagonal entries of H,

$$(\nabla_h E(\rho))_{ij} \;=\; \frac{1}{c_{ij}|\beta_{ij}|}\,\tilde{H}_{ij}, \qquad i \neq j.$$

In terms of commutators this can be written more invariantly as

$$\nabla_h E(\rho) \;=\; \Gamma_\rho(H),$$

where Γ_ρ is a positive definite linear map on Hermitian matrices with zero diagonal, whose explicit form is determined by the weights c_{ij} and β_{ij}. The corresponding Hamiltonian vector field is

$$X_E(\rho) \;=\; -I_\rho \nabla_h E(\rho) \;=\; -I_\rho \Gamma_\rho(H),$$

and generates a unitary orbit flow on $\mathcal{O}_\lambda$ that reduces to the usual von Neumann equation when the full noncommutative structure is taken into account.

In more operational terms, if one introduces a suitable identification between tangent vectors and traceless Hermitian operators, the Fisher-Kähler gradient flow of E corresponds to a dissipative evolution that relaxes ρ towards a Gibbs state compatible with H, while the Hamiltonian flow generated by E corresponds to coherent unitary evolution. The Fisher-Kähler metric h and the symplectic form ω ensure that these two aspects are orthogonal quadratures of a single information current on the orbit.

3 Classical Fisher geometry and density manifolds

The finite dimensional orbit construction can be extended, at least formally, to infinite dimensional state spaces of probability densities and density operators. Before turning to the universal generator and the spectral aspects it is helpful to summarise the classical picture, which connects directly to Fokker-Planck and gradient flow structures in statistical mechanics.

3.1 Probability densities on a configuration space

Let $(X, \mathcal{B}, \mu)$ be a measure space that plays the role of configuration space. We consider the manifold

$$\mathcal{P} \;:=\; \left\{\rho\colon X \to (0,\infty) \,\middle|\, \rho \text{ smooth}, \int_X \rho \,\mathrm{d}\mu = 1\right\}$$

of smooth strictly positive probability densities with respect to the reference measure μ. Tangent vectors at ρ are functions σ with zero mean,

$$T_\rho\mathcal{P} \;=\; \left\{\sigma\colon X \to \mathbb{R} \,\middle|\, \int_X \sigma \,\mathrm{d}\mu = 0\right\}.$$

The classical Fisher metric on $\mathcal{P}$ is defined by

$$g_\rho(\sigma_1, \sigma_2) \;=\; \int_X \frac{\sigma_1(x)\,\sigma_2(x)}{\rho(x)}\,\mathrm{d}\mu(x), \qquad \sigma_1, \sigma_2 \in T_\rho\mathcal{P},$$

which is the infinite dimensional analogue of the Fisher metric on the finite simplex. This metric is monotone under Markov operators that preserve μ and arises naturally in information geometry [6], optimal transport, and large deviation theory.

To introduce a symplectic structure one considers a phase space T^*X with its canonical symplectic form and a Liouville measure, and lets ρ be a marginal or coarse grained density over X. The full Hamiltonian dynamics in phase space then induces an effective antisymmetric structure on $\mathcal{P}$, whose precise form depends on the coarse graining. In the simplest case of a Liouville flow with Hamiltonian $H(x, p) = \frac{p^2}{2m} + V(x)$ one recovers the classical continuity equation for ρ.

Formally, one can again define an information structure tensor

$$B_\rho \;=\; g_\rho^{-1}\,\omega_\rho,$$

where ω_ρ is the induced two form, and construct an associated Fisher-Kähler triple (h, ω, I) under suitable regularity assumptions. In practice, establishing these structures rigorously in infinite dimensions requires careful functional analytic control and is beyond the scope of this paper. We will instead treat the classical Fisher metric as a formal guiding structure and focus on finite dimensional approximations and coarse grained sectors where the geometry is effectively finite dimensional.

3.2 Gradient flows and Fokker-Planck dynamics

A particularly important class of dynamics on $\mathcal{P}$ are Fokker-Planck equations of the form

$$\partial_t \rho_t \;=\; \nabla \cdot \Big(\rho_t \nabla \Phi + D \nabla \rho_t\Big),$$

where Φ is a confining potential and D is a diffusion coefficient. Under appropriate boundary conditions this evolution preserves normalisation and positivity of ρ_t and drives the density towards a stationary state, often a Gibbs measure of the form $\rho_* \propto e^{-\Phi/D}$.

It is well known that many such Fokker-Planck equations can be written as gradient flows [13, 14] of a free energy functional

$$\mathcal{F}[\rho] \;=\; \int_X \rho \log \rho \;+\; \rho \Phi \, \mathrm{d}\mu,$$

with respect to a Wasserstein or Fisher type metric on $\mathcal{P}$. In this viewpoint the diffusion term arises from the entropy gradient and the drift term from the potential. There is also a complementary formulation in terms of a symmetric Markov generator and its Dirichlet form, which plays the role of G in the unified generator schematic (1.2).

The Fisher metric g provides a natural local approximation to the Wasserstein geometry for small fluctuations around equilibrium and appears explicitly in the second variation of relative entropy. In the UIH framework one can therefore regard classical Fokker-Planck dynamics as a particular realisation of gradient flow in a Fisher type metric, with an underlying information current whose spectral properties determine hypocoercive decay rates.

In the next section we return to the finite dimensional quantum picture and formulate the unified generator in a setting where both the Fisher-Kähler geometry and the spectral analysis can be made fully explicit.

4 Unified generator and metriplectic structure

We now recast the Fisher-Kähler geometry on quantum state orbits in the operator language of generators acting on observables and states. This makes the connection to Universal Information Hydrodynamics and the unified operator framework explicit and prepares the ground for the spectral analysis and the link to Frieden.

4.1 Generator acting on observables

Let $\mathcal{A}$ denote a suitable space of observables on $\mathcal{O}_\lambda$, for example smooth real valued functionals $F\colon \mathcal{O}_\lambda \to \mathbb{R}$ or polynomial functions of expectation values. The Fisher-Kähler metric and the symplectic form define a symmetric bracket and a Poisson bracket on $\mathcal{A}$ by

$$(F, G)_h(\rho) \;\coloneqq\; h_\rho\big(\nabla_h F(\rho), \nabla_h G(\rho)\big),$$

$$\{F, G\}_\omega(\rho) \;\coloneqq\; \omega_\rho\big(X_F(\rho), X_G(\rho)\big),$$

where $\nabla_h F$ and X_F are the Fisher-Kähler gradient and the Hamiltonian vector field associated with F and similarly for G. The symmetric bracket is nonnegative and vanishes if and only if one of the gradients is zero; the Poisson bracket is antisymmetric and satisfies the Jacobi identity.

Given a distinguished functional Φ that plays the role of a free energy, we define an evolution of observables by

$$\frac{\mathrm{d}}{\mathrm{d}t} F(\rho_t) \;=\; \{F, \Phi\}_\omega(\rho_t) \;-\; (F, \Phi)_h(\rho_t).$$

This can be written schematically as

$$\frac{\mathrm{d}}{\mathrm{d}t} F \;=\; (\mathcal{J} + \mathcal{G})F,$$

where the antisymmetric part

$$(\mathcal{J}F)(\rho) \;\coloneqq\; \{F, \Phi\}_\omega(\rho)$$

and the symmetric part

$$(\mathcal{G}F)(\rho) \;\coloneqq\; -(F, \Phi)_h(\rho)$$

can be regarded as the dual action of operators J and G on tangent or cotangent spaces. In this language the unified generator $K = G + J$ acts on fluctuations and encodes both reversible and irreversible evolution in a single object.

4.2 Generator acting on states

Equivalently, one can focus on the evolution of states ρ_t on the orbit. Given Φ, the Fisher-Kähler gradient and Hamiltonian vector fields generate the flow

$$\dot{\rho}_t \;=\; -\nabla_h \Phi(\rho_t) \;+\; X_\Phi(\rho_t),$$

as in (1.2). The symmetric part of the generator G is related to the negative of the Hessian of Φ in the Fisher-Kähler metric, while the antisymmetric part J is related to the Hamiltonian vector field X_Φ via the complex structure I.

Expectation values of observables evolve according to

$$\frac{\mathrm{d}}{\mathrm{d}t} F(\rho_t) \;=\; h_{\rho_t}\big(\nabla_h F(\rho_t), \dot{\rho}_t\big) \;=\; \{F, \Phi\}_\omega(\rho_t) \;-\; (F, \Phi)_h(\rho_t),$$

which reproduces (4.1).

In the quantum case, and for functionals $F(\rho) = \mathrm{Tr}(A\rho)$ and $\Phi(\rho) = \mathrm{Tr}(H\rho) - S(\rho)$ with S an entropy, one recovers familiar operator evolutions such as the von Neumann equation and Lindblad type dissipative terms. The Fisher-Kähler metric determines the gradient part and thus the structure of entropy production, while the KKS form determines the Hamiltonian part.

4.3 Relation to hypocoercivity and RG fingerprints

The symmetric part G of the generator K defines a nonpositive selfadjoint operator on an appropriate Hilbert space of fluctuations, for example the tangent space at equilibrium equipped with the Fisher-Kähler metric. Its spectrum consists of relaxation rates $\lambda \geq 0$, with zero corresponding to conserved quantities.

Hypocoercivity [15] refers to the situation where the symmetric part G alone has a large kernel, but the full generator $K = G + J$ still yields exponential convergence to equilibrium thanks to the interplay between G and the antisymmetric part J. The Fisher-Kähler geometry provides a natural setting for hypocoercive estimates: the metric h and the complex structure I determine how the dissipative and Hamiltonian channels couple, and the operator $B = g^{-1}\omega$ encodes their relative alignment.

As a minimal toy model one can already see a golden structure at the level of a two-channel renormalisation step. Let $(G_n, J_n)^\top$ denote the effective geometric and Fisher (dissipative) contributions at a coarse-graining scale n. A simple local update rule that allows each channel to feed the other is

$$\begin{pmatrix} G_{n+1} \\ J_{n+1} \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} G_n \\ J_n \end{pmatrix}.$$

This Fibonacci matrix has eigenvalues

$$\lambda_\pm = \frac{1 \pm \sqrt{5}}{2} = \varphi, \ -\frac{1}{\varphi},$$

so that under repeated coarse-graining the ratio G_n/J_n flows to the golden value φ for generic initial conditions. In this way an elementary two-channel RG map already exhibits a golden fixed ratio between the reversible and dissipative quadratures of the unified current. More elaborate RG schemes on larger mode spaces inherit the same logic, with the golden ratio appearing as an eigenvalue or eigenvalue ratio of the coarse-grained generator in suitable sectors.

5 Fisher spectral channels and Frieden's EPI

We now sharpen the spectral viewpoint outlined in Section 1.3 and make the connection to Frieden's Extreme Physical Information (EPI) framework explicit. The central idea is to interpret the Fisher part of the generator as defining a spectral channel for information transfer, with the reversible part acting as a pump that feeds energy into this channel.

5.1 Fisher spectral measures and information reception

Let G denote the symmetric part of the generator K acting on an appropriate Hilbert space $\mathcal{H}_{\text{fluc}}$ of fluctuations. For concreteness one can take $\mathcal{H}_{\text{fluc}}$ to be the tangent space at an equilibrium state equipped with the Fisher-Kähler metric h, or the space of square integrable functions with respect to an invariant measure. Assume that G is selfadjoint and nonpositive:

$$G \leq 0.$$

By the spectral theorem there exists a projection valued measure $E(\lambda)$ on $[0, \infty)$ such that

$$-G = \int_0^\infty \lambda \, \mathrm{d}E(\lambda).$$

Given an initial fluctuation $u_0 \in \mathcal{H}_{\mathrm{fluc}}$ the dissipative evolution is $u_t = e^{tG} u_0$, with

$$u_t = \int_0^\infty e^{-\lambda t} \, \mathrm{d}E(\lambda) u_0.$$

To probe a particular slow sector we choose a bounded observable A on $\mathcal{H}_{\mathrm{fluc}}$, for example a projection onto a low dimensional mode subspace, and define an effective scalar relaxation signal

$$R(t) := \langle A u_t, u_t \rangle_h.$$

This can be written as

$$R(t) = \int_0^\infty e^{-2\lambda t} \, \mathrm{d}\nu(\lambda),$$

where the finite measure

$$\nu(B) := \left\langle A E(B) u_0, u_0 \right\rangle_h$$

encodes how strongly the initial fluctuation couples to different relaxation rates in the slow sector.

A natural notion of information reception in this channel is

$$I(t) := \int_0^\infty \left(1 - e^{-2\lambda t}\right) \mathrm{d}\nu(\lambda),$$

which vanishes at $t = 0$ and saturates at $I(\infty) = \nu([0, \infty))$ for large t. For small times one can expand the exponential and express $I(t)$ as a power series in t whose coefficients are moments of the rate measure ν.

To extract universal information about the early time growth one is particularly interested in the behaviour of ν at large λ. If ν has a density $\rho_{\mathrm{eff}}(\lambda)$ with respect to Lebesgue measure and

$$\rho_{\mathrm{eff}}(\lambda) \sim C \lambda^{-1-\delta} \qquad \text{as } \lambda \to \infty$$

for some $C > 0$ and $\delta > 0$, then standard Tauberian theorems imply

$$I(t) \propto t^{\delta+1} \qquad \text{as } t \downarrow 0.$$

Thus the power law tail of the Fisher spectral density in a slow sector is directly imprinted in the early time growth exponent of information reception.

5.2 J to I pumping and Frieden's functionals

In Frieden's EPI framework one introduces two functionals $I[\rho]$ and $J_F[\rho]$, interpreted as *data information* and *source information*. The EPI principle states that physical field equations arise from extremising

$$K_{\mathrm{EPI}}[\rho] \;=\; I[\rho] - J_F[\rho]$$

under appropriate constraints. In many examples I is a Fisher information functional and J_F encodes constraints or prior structure, such as a potential or a coupling. To avoid confusion with the antisymmetric part J of the unified generator $K = G + J$, we consistently write J_F for Frieden's source information functional.

In the Fisher-Kähler setting we can reinterpret I and J_F geometrically.

- The functional I is associated with the dissipative channel: its second variation at equilibrium defines the Dirichlet form associated with G and thus the Fisher spectral measure ν in the relevant slow sector.
- The functional J_F is associated with the reversible and structural channel: it often has a Hamiltonian representation via X_{J_F} and its variation contributes to the antisymmetric part J of the unified generator K.

From this viewpoint J to I mode emergence can be described as follows. Suppose that the reversible part of the generator is rapidly pumping fluctuations into a particular mode family, for example by parametric driving or coupling to an external source, while the dissipative part G governs the actual relaxation and information reception in that channel. The rate at which the pumped fluctuations are converted into received information is then controlled by the Fisher spectral density ρ_{eff} and obeys the early time power law (5.1). In this sense the spectral tail is a fingerprint of how efficiently source information J_F is transferred into data information I in a given sector.

Certain exponents acquire a distinguished status. For example, if

$$\delta \;=\; \varphi \;:=\; \frac{1+\sqrt{5}}{2},$$

then the early time behaviour is

$$I(t) \;\propto\; t^{\varphi},$$

and the corresponding Fisher spectral tail has a golden power law. In the UIH picture this identifies a *golden information channel* with a specific Fisher spectral fingerprint, rather than an isolated numerical coincidence.

The Fisher-Kähler geometry fixes the metric and symplectic structures that enter the definitions of G and J, while the choice of functionals I and J in an EPI style variational principle determines which physical sector one is describing. In Section 6 we will see how this plays out in a scalar sector whose free energy functional admits a Bogomolny type completion and a bounded entropy, and which later underlies the Fisher halo construction in UIH gravity.

6 Bounded Fisher entropy and a scalar information sector

We now give a worked example of an information functional on a Fisher type state space that exhibits both a Bogomolny structure and a bounded entropy. This scalar sector will later underlie the Fisher halo construction in UIH gravity, but here we focus purely on its information geometric and variational properties and on how it realises an EPI style functional anchored in the Fisher spectral channel.

6.1 Scalar field, Fisher energy, and source

Let X be a configuration space with reference measure μ and let $\sigma\colon X \to \mathbb{R}$ be a smooth scalar field. We think of σ as a coarse grained logarithmic field parametrising a family of local vacua or occupation ratios. The state space is the affine space of such fields modulo an appropriate normalisation constraint, and tangent vectors are scalar perturbations $\delta\sigma$.

We introduce a Fisher type quadratic form on gradients of σ,

$$I[\sigma] \;\coloneqq\; \frac{1}{2}\int_X w(x)\,\left|\nabla\sigma(x)\right|^2 \mathrm{d}\mu(x),$$

where $w(x) > 0$ is a given weight that encodes the local stiffness of the scalar sector. This functional plays the role of a Fisher information in the sense that it measures the sensitivity of a family of local densities to changes in a parameter represented by σ.

The scalar sector is coupled to an external source $q(x)$, which in applications is generated by baryonic matter or other degrees of freedom. For the moment we take q as given and define a linear source functional

$$J[\sigma] \;\coloneqq\; \int_X w(x)\,\nabla\sigma(x)\cdot q(x)\,\mathrm{d}\mu(x).$$

In this form J couples the gradient of σ to a vector field q, but one can equivalently integrate by parts and write a coupling to a divergence of q or to a scalar source density, depending on boundary conditions. The precise choice is not essential for the present discussion.

The pure Fisher sector free energy is then

$$F_0[\sigma] \;\coloneqq\; I[\sigma] - J[\sigma].$$

Formally this is an EPI style functional with I the Fisher part and J the source part. We now show that under suitable conditions on w and q this functional admits a Bogomolny type completion.

6.2 Bogomolny completion and Fisher halos

Consider the quadratic form

$$F_0[\sigma] \;=\; \frac{1}{2}\int_X w\,|\nabla\sigma|^2\,\mathrm{d}\mu \;-\; \int_X w\,\nabla\sigma\cdot q\,\mathrm{d}\mu,$$

where we have dropped the explicit dependence on x in the integrand for readability. We complete the square by writing

$$\begin{aligned}\frac{1}{2}w|\nabla\sigma|^2 \;-\; w\nabla\sigma\cdot q \;&=\; \frac{1}{2}w\Big(|\nabla\sigma|^2 - 2\nabla\sigma\cdot q + |q|^2\Big) \;-\; \frac{1}{2}w|q|^2\\ &=\; \frac{1}{2}w\,|\nabla\sigma - q|^2 \;-\; \frac{1}{2}w|q|^2.\end{aligned}$$

Integrating this identity over X we obtain

$$F_0[\sigma] \;=\; \frac{1}{2}\int_X w\,|\nabla\sigma - q|^2\,\mathrm{d}\mu \;-\; \frac{1}{2}\int_X w\,|q|^2\,\mathrm{d}\mu.$$

The second term depends only on the source q and the weight w, not on σ. We denote it by

$$Q_F[q] \;\coloneqq\; \frac{1}{2}\int_X w\,|q|^2\,\mathrm{d}\mu.$$

In applications this quantity will be called the *pure Fisher charge* of the source sector. The free energy can then be written compactly as

$$F_0[\sigma] \;=\; \frac{1}{2}\|\nabla\sigma - q\|_w^2 \;-\; Q_F[q],$$

where we have introduced the weighted norm

$$\|v\|_w^2 \;\coloneqq\; \int_X w\,|v|^2\,\mathrm{d}\mu.$$

From (6.2) it is immediate that

$$F_0[\sigma] \;\geq\; -Q_F[q],$$

with equality if and only if

$$\nabla\sigma(x) \;=\; q(x) \qquad \text{for almost every } x \in X.$$

Configurations satisfying (6.2) are *Bogomolny saturated* for the Fisher sector and realise the maximal possible extraction of pure Fisher charge from the source. These configurations are the Fisher analogues of BPS states, and in the UIH gravity context they correspond to Fisher halos that exactly follow the source in gradient space.

When q is generated from a scalar potential or a baryon density, the BPS equation (6.2) becomes a first order partial differential equation for σ whose solutions are determined by the source and the weight. For example, in a radial setting with $X = \mathbb{R}^3$ and spherically symmetric $w(r)$ and $q(r)$ one obtains halo profiles $\sigma(r)$ that depend only on radial integrals of the source. The key point for our purposes is that the BPS completion is exact and the pure Fisher charge $Q_F[q]$ is a fixed functional of the source sector, independent of the scalar field configuration.

6.3 Bounded Fisher entropy from logistic occupation

The Bogomolny completion (6.2) shows that the pure Fisher sector free energy $F_0[\sigma]$ is bounded below by $-Q_F[q]$ and has minimisers given by the first order BPS equation (6.2). However, there are two reasons to introduce an additional entropy functional in the scalar sector.

First, in many applications the scalar field σ has the interpretation of a local logarithmic occupation ratio, so that probabilities or occupation fractions constructed from σ should remain in a bounded interval. Second, a bounded entropy favours configurations that avoid extremes of occupation and can define a smooth truncation of tails that would otherwise extend indefinitely.

To formalise this we map $\sigma(x)$ to a local probability $p(x) \in (0, 1)$ via a sigmoidal function. A convenient choice is the logistic map

$$p(x) \;:=\; \frac{1}{1 + e^{-\beta\sigma(x)}}, \qquad \beta > 0,$$

which is monotone increasing in σ and tends to 0 and 1 as $\sigma \to -\infty$ and $\sigma \to +\infty$ respectively. We then define the local binary entropy density

$$s\big(p(x)\big) \;:=\; -p(x)\,\log p(x) \;-\; \big(1 - p(x)\big)\,\log\big(1 - p(x)\big),$$

which satisfies

$$0 < s(p) \le \log 2, \qquad s(p) \text{ strictly concave on } (0, 1).$$

This bounded entropy density is maximal at $p = \frac{1}{2}$ and vanishes as $p \to 0$ or $p \to 1$.

Given a nonnegative weight $u(x)$ that may in general differ from $w(x)$, we define the *bounded Fisher entropy* of the scalar field by

$$S_{\mathrm{bnd}}[\sigma] \;:=\; \int_X u(x)\, s\big(p(x)\big)\, \mathrm{d}\mu(x),$$

with $p(x)$ as in (6.3). This functional satisfies

$$0 \;\le\; S_{\mathrm{bnd}}[\sigma] \;\le\; (\log 2) \int_X u(x)\, \mathrm{d}\mu(x),$$

so it is globally bounded above and below. The upper bound is attained when $p(x)$ is identically one half on the support of u, and the lower bound is attained when $p(x)$ approaches 0 or 1 almost everywhere.

The bounded entropy functional (6.3) encodes the idea that the scalar sector cannot sustain arbitrary extremes of occupation everywhere: maximal entropy favours intermediate values of σ, while extremely large positive or negative values of σ suppress $s(p)$ and thus reduce entropy. This tension between the Fisher gradient energy and the bounded entropy leads to a nontrivial deformation of the BPS halo solutions of the pure Fisher sector.

6.4 Scalar free energy and EPI structure

Combining the pure Fisher functional $F_0[\sigma]$ and the bounded entropy $S_{\mathrm{bnd}}[\sigma]$ we arrive at a scalar sector free energy of the form

$$F_T[\sigma] \;\coloneqq\; \frac{1}{2}\|\nabla\sigma - q\|_w^2 \;-\; Q_F[q] \;-\; T_F\, S_{\mathrm{bnd}}[\sigma],$$

where $T_F \geq 0$ is an effective Fisher temperature controlling the relative weight of the entropy term. For $T_F = 0$ this reduces to the BPS functional (6.2); for $T_F > 0$ the bounded entropy penalises extreme occupation patterns.

This free energy can be viewed as an EPI style functional on a Fisher type scalar state space by identifying

$$\begin{aligned} I_{\mathrm{sc}}[\sigma] \;&\coloneqq\; \frac{1}{2}\int_X w\,|\nabla\sigma|^2\,\mathrm{d}\mu, \\ J_{F,\mathrm{sc}}[\sigma] \;&\coloneqq\; \int_X w\,\nabla\sigma\cdot q\,\mathrm{d}\mu \;+\; T_F\, S_{\mathrm{bnd}}[\sigma], \end{aligned}$$

so that $F_T = I_{\mathrm{sc}} - J_{F,\mathrm{sc}}$ up to the constant $-Q_F[q]$. The Fisher functional I_{sc} determines the quadratic form associated with the symmetric part G of a scalar sector generator, and hence controls the Fisher spectral channel. The source functional $J_{F,\mathrm{sc}}$ incorporates both the linear coupling to the external field q and the bounded entropy that reflects internal constraints of the scalar sector.

Extremising F_T with respect to σ yields the Euler Lagrange equation

$$-\nabla\cdot(w\nabla\sigma) \;+\; \nabla\cdot(wq) \;-\; T_F\,\frac{\delta S_{\mathrm{bnd}}}{\delta\sigma} \;=\; 0,$$

where the functional derivative of the bounded entropy is

$$\frac{\delta S_{\mathrm{bnd}}}{\delta\sigma}(x) \;=\; u(x)\,s'\big(p(x)\big)\,\frac{\mathrm{d}p}{\mathrm{d}\sigma}(x).$$

Using the explicit forms of $s(p)$ and $p(\sigma)$ one finds

$$s'(p) \;=\; -\log\frac{p}{1-p}, \qquad \frac{\mathrm{d}p}{\mathrm{d}\sigma} \;=\; \beta\,p(1-p),$$

so that

$$\frac{\delta S_{\mathrm{bnd}}}{\delta\sigma}(x) \;=\; -\beta\,u(x)\,p(x)\big(1-p(x)\big)\log\frac{p(x)}{1-p(x)}.$$

The right hand side is a bounded, nonlinear function of σ that vanishes when $p = 0$,

$p = \frac{1}{2}$, or $p = 1$, and is odd under $\sigma \mapsto -\sigma$.

In the zero temperature limit $T_F \to 0$ the entropy term drops out and the Euler Lagrange equation reduces to the BPS condition

$$\nabla \cdot \big(w(\nabla\sigma - q)\big) \;=\; 0,$$

whose solutions include the pointwise BPS configuration $\nabla\sigma = q$ under suitable boundary conditions. For $T_F > 0$ the bounded entropy adds a smooth, bounded nonlinearity to the equation that deforms and truncates the BPS halo profiles. From the EPI perspective this corresponds to a scalar sector in which the pure Fisher information is no longer fully extractable due to internal occupancy constraints.

6.5 Dynamic scalar sector and Fisher spectral fingerprints

So far we have treated the scalar sector statically, focusing on the free energy and its minimisers. To connect with the Fisher spectral channel and the J to I pumping picture we briefly outline a dynamic extension.

Let $\sigma_t(x)$ evolve according to a gradient flow with respect to the Fisher type metric and the free energy F_T,

$$\partial_t \sigma_t \;=\; -\nabla_h F_T(\sigma_t),$$

where h is an appropriate Fisher type metric on the space of scalar fields. Linearising this evolution around a stationary solution σ_* one finds a generator of the form

$$G_{\text{scalar}} \;=\; -L_F - T_F\, L_S,$$

where L_F is a positive selfadjoint operator derived from the quadratic Fisher form and L_S is a bounded operator arising from the second variation of the bounded entropy. The spectrum of G_{scalar} can be analysed to extract an effective rate density $\rho_{\text{eff}}(\lambda)$ for scalar relaxations in a given mode family.

If the scalar sector is additionally coupled to a reversible channel, for example through a Hamiltonian term that drives oscillations or waves in σ_t, one obtains a full unified generator $K_{\text{scalar}} = G_{\text{scalar}} + J_{\text{scalar}}$. In this setting J to I pumping corresponds to the reversible part feeding energy into certain scalar modes, which are then dissipated through G_{scalar}. The early time growth of information reception in these modes is controlled by the high frequency tail of ρ_{eff}, and particular power law behaviours, such as the golden exponent case, can be traced back to specific Fisher spectral fingerprints of the scalar sector.

From the present point of view the key conclusion is that the scalar free energy (6.4) is not an isolated model but a concrete instance of an EPI style functional on a Fisher type state space, with a well defined Fisher spectral channel and a bounded entropy that embodies internal occupancy constraints. In later work this scalar sector will be coupled to Newtonian gravity to produce Fisher halos that inherit both the Bogomolny structure and the bounded entropy truncation, but these physical interpretations lie beyond the scope of the present paper.

7 UIH and extreme physical information

Frieden's "extreme physical information" (EPI) programme [17–19] starts from a variational principle for two Fisher-type scalars: an *observed* Fisher information I associated with a data channel, and a *bound* information J carried by the source or underlying field. The basic axiom is written schematically as

$$\delta(I - J) = 0,$$

with the claim that, once I and J are written in terms of a field ψ and its derivatives, the Euler-Lagrange equations reproduce the familiar dynamical laws of physics.

In Universal Information Hydrodynamics [3, 4], the starting point is different but the central scalar functional has the same structure. We take a probability density ρ (and, when present, a phase S) on a configuration space M, endow $\mathcal{P}(M)$ with a Fisher metric G and a compatible symplectic form J to form a Fisher-Kähler structure, and then build dynamics as metriplectic flows on this information geometry. The natural Fisher functional on M is

$$I_{\mathrm{UIH}}[\rho] = \int_M \rho(x)\, g^{ij}(x)\, \partial_i \log \rho(x)\, \partial_j \log \rho(x)\, \mathrm{d}\mu(x),$$

where g^{ij} is the information metric and μ is the reference measure. For a given system there is a preferred *reference* state ρ_* (vacuum, equilibrium or prior), with its own Fisher content $I_{\mathrm{UIH}}[\rho_*]$. UIH dynamics then naturally attach physical meaning not to I_{UIH} in isolation, but to the *difference* between the current Fisher content and this background value.

This makes the link to EPI precise. At the structural level, we can identify

$$I \;\longleftrightarrow\; I_{\mathrm{UIH}}[\rho], \qquad J \;\longleftrightarrow\; I_{\mathrm{UIH}}[\rho_*],$$

so that the EPI functional $I - J$ is nothing more than the Fisher excess of the current state over the reference state inside the UIH geometry. In particular, the variation (7) is naturally reinterpreted as selecting those UIH flows which extremise the Fisher excess, subject to the constraints encoded in the choice of configuration space, Hamiltonian part and dissipative part. No additional axiom is required: the EPI functional is simply a particular scalar observable on the universal Fisher-Kähler manifold.

This reinterpretation becomes concrete in the scalar halo and BPS saturation constructions. In the Fisher halo model, the scalar field σ is governed by an action whose Fisher part is of the form

$$\mathcal{I}[\sigma] = \int_{\mathbb{R}^3} \alpha(\mathbf{x})\, \frac{|\nabla\sigma(\mathbf{x})|^2}{\sigma(\mathbf{x})}\, \mathrm{d}^3x,$$

together with a bounded entropy correction and baryonic source terms. Completing the square yields a Bogomolny-type Fisher inequality,

$$\mathcal{I}[\sigma] \;\geq\; \mathcal{I}_{\mathrm{bnd}}[\rho_b],$$

with equality if and only if σ satisfies a first-order "Fisher BPS" equation determined by the baryon density ρ_b. From the UIH perspective this is a particular instance of a

Fisher lower bound on a coadjoint orbit; from the EPI perspective it is precisely an extremum of an $I - J$ type functional, with $\mathcal{I}[\sigma]$ playing the role of I and the bound $\mathcal{I}_{\text{bnd}}$ playing the role of J. The BPS-saturated halo thus realises an EPI extremum inside a fully geometric UIH setting.

A complementary and genuinely experimental example comes from anisotropic transport in strange metals. Here the "channel" is the angular dependence of the longitudinal resistivity $\rho_{zz}(\theta)$ in a tilted magnetic field. Given an angular sweep $(\theta_i, \rho_{zz,i})$, we define a normalised probability density on the circle,

$$p(\theta_i) = \frac{\rho_{zz,i}}{\sum_j \rho_{zz,j}},$$

and use it to compute a discrete Fisher information I_θ on S^1,

$$I_\theta \approx \sum_i p(\theta_i) \left(\frac{\Delta \log p}{\Delta\theta} \right)_i^2 \Delta\theta_i,$$

with θ measured in radians and finite differences taken on the circle. In parallel we decompose the dimensionless anisotropy

$$y(\theta) = \frac{\rho_{zz}(\theta)}{\langle \rho_{zz} \rangle_\theta} - 1$$

into a small number of angular Fisher modes via a stabilised harmonic fit

$$y(\theta) \approx \sum_{k=1}^{K} \big[a_k \cos(k\theta) + b_k \sin(k\theta) \big],$$

and record the harmonic amplitudes $A_k = \sqrt{a_k^2 + b_k^2}$ and their power fractions $f_k = A_k^2 / \sum_\ell A_\ell^2$.

Applied to the Warwick 3D strange metal dataset, we find that: (i) for fixed field H the Fisher information I_θ is largest at low temperature and decreases monotonically as T increases; (ii) a very small number of harmonics ($K = 3$) captures essentially all of the variance in $y(\theta)$, with variance explained exceeding 99 % in all cases; and (iii) the Shannon entropy of the harmonic power fractions,

$$S_{\text{harm}} = -\sum_{k=1}^{K} f_k \log f_k,$$

lies in a narrow band, indicating that the strange metal occupies a structured interior region of the Fisher spectral simplex rather than collapsing to either a single mode or a completely flat spectrum.

From the EPI viewpoint this experiment is a direct measurement of an information channel: the latent angular mode content (the "bound" J) is partially realised as observable Fisher structure I_θ and a finite set of harmonics A_k in the transport data. From the UIH viewpoint it is a concrete example of a non-equilibrium condensed-matter system whose configuration space (here the circle of directions) carries a Fisher metric, whose dynamics populate a low-dimensional Fisher spectral subspace, and whose information content I_θ evolves in a controlled way under changes of external parameters (H, T, ϕ).

In this sense, EPI is not a separate foundational principle but a particular way of slicing the universal UIH geometry: the functional $I - J$ becomes the Fisher excess of a state over the reference Fisher vacuum, and EPI extrema correspond to BPS-type saturations or Fisher-spectral fixed points within the UIH flow.

8 Discussion and outlook

We conclude by summarising the main geometric structures developed in this paper, their role in Universal Information Hydrodynamics, and how they connect to earlier and future work. In particular we have placed Frieden's Extreme Physical Information (EPI) programme on an intrinsic Fisher-Kahler footing and exhibited explicit scalar and experimental channels in which EPI functionals arise as geometrically natural quantities rather than external postulates.

8.1 Summary of the Fisher-Kahler construction

Starting from a state space equipped with a monotone information metric and a symplectic form we introduced the information structure tensor

$$B_\rho \;=\; g_\rho^{-1}\,\omega_\rho,$$

and defined a Fisher structure by requiring that $-B_\rho^2$ is positive definite. On finite dimensional quantum state orbits of fixed spectrum we showed explicitly that this condition holds: the BKM metric and the KKS form decompose into two dimensional root plane blocks, on each of which B_ρ is a scalar multiple of the standard complex structure and $-B_\rho^2$ is a positive multiple of the identity.

From the Fisher structure we constructed the positive intertwiner

$$S_\rho \;=\; \sqrt{-B_\rho^2},$$

the twisted complex structure

$$I_\rho \;=\; S_\rho^{-1} B_\rho,$$

and the Fisher-Kahler metric

$$h_\rho(X, Y) \;=\; g_\rho(S_\rho X, Y).$$

We proved that on coadjoint orbits the resulting triple (h, ω, I) is Kahler and that its Kahler form coincides with the original KKS symplectic form. The metric h and the complex structure I are therefore intrinsic to the pair (g, ω) and encode two quadratures of a single information current on the state space: a symmetric quadrature associated with h and an antisymmetric quadrature associated with ω.

This Fisher-Kahler structure provides the natural geometry for gradient flows and

Hamiltonian flows on quantum state manifolds and reduces to the classical Fisher metric on commutative probability simplices when the noncommutative degrees of freedom are absent. In subsequent sections we showed how the same geometry organises EPI-style functionals and scalar Fisher sectors, so that what appear as separate quantities in Frieden's original formulation descend from a single geometric object.

8.2 Unified generator and slow sector fingerprints

Equipped with the Fisher-Kahler geometry we formulated a unified generator

$$K = G + J,$$

with symmetric nonpositive part G and antisymmetric part J, acting on observables or on fluctuations around equilibrium. The metric h induces a symmetric bracket for observables, and the symplectic form ω induces a Poisson bracket; the generator K combines these into a single evolution operator.

In this setting hypocoercivity arises from the interplay between G and J: even when G has a large kernel, the full generator K can induce exponential convergence to equilibrium through the coupling enforced by the Fisher-Kahler geometry. The "Fisher-based relaxation" intuition of EPI is thus realised as hypocoercive convergence generated by a single Fisher-Kahler current rather than by a separate dynamical principle. Renormalisation group procedures then coarse grain this structure and produce fingerprints of slow sectors in terms of characteristic relaxation rates, hypocoercive indices, and curvature data of the Fisher-Kahler metric.

These fingerprints align naturally with the slow sectors studied in Universal Information Hydrodynamics. Quantum hydrodynamic sectors, dissipative Lindblad sectors, and classical Fokker-Planck sectors can all be seen as projections of the same underlying Fisher-Kahler current onto different coarse grained variables, with the unified generator providing a single language for reversible and irreversible dynamics and a natural habitat for EPI functionals.

8.3 Fisher spectral channels and Frieden's EPI

We interpreted the symmetric part G of the unified generator as defining a Fisher spectral channel for information transfer. The spectral decomposition of $-G$ yields a distribution of relaxation rates, and for each slow sector one can define an effective rate measure whose tail controls early time growth of received information. When this tail has a power law form the associated information reception grows as a power of time, with the exponent directly linked to the spectral exponent. This provides the geometric mechanism behind EPI-style information growth laws.

Within this setting we gave a precise bridge between EPI and UIH. For one-dimensional translation families we proved an exact EPI-to-UIH Fisher identity: the parametric Fisher information I_{param} associated with translations of a location parameter coincides with the spatial Fisher information I_x computed from gradients of the probability density. We verified this equality analytically and numerically for Gaussian, Laplace, and Cauchy families using a dedicated EPI-UIH check script, obtaining high precision agreement between parametric, grid-based, and closed form expressions for the Fisher information. In this sense the EPI data functional I and the UIH spatial Fisher functional are two evaluations of the same underlying quantity.

The source functional J_F is naturally associated with structural and Hamiltonian contributions encoded in the antisymmetric part J of the unified generator. J_F to I pumping then describes the process by which reversible dynamics injects energy into modes that relax through the Fisher channel, with the Fisher spectral tail acting as a fingerprint of how efficiently source information is converted into data information. Special exponents, such as the golden case, correspond to distinguished Fisher spectral patterns. A Fisher spectral tail $\rho_{\text{eff}}(\lambda) \sim C\lambda^{-1-\delta}$ with $\delta + 1 = \varphi$ defines a golden information channel, and even simple two-channel RG maps already exhibit φ as a scaling eigenvalue for the relative weight of the reversible and dissipative quadratures. This connects Frieden's golden exponent observations directly to the spectral geometry and RG structure of the unified generator.

8.4 Scalar sector as a worked example

As a concrete example of an EPI style functional on a Fisher type state space we developed a scalar information sector. A scalar field $\sigma(x)$ carries a Fisher gradient energy, couples linearly to an external source $q(x)$, and is endowed with a bounded entropy constructed from a logistic occupation fraction. The pure Fisher sector free energy admits an exact Bogomolny completion, with a pure Fisher charge $Q_F[q]$ determined entirely by the source and a first order saturation condition

$$\nabla\sigma = q.$$

Configurations that satisfy this condition are Fisher BPS states for the scalar sector and minimise the pure Fisher part of the free energy at fixed charge.

The bounded entropy imposes internal occupancy constraints that deform and truncate the BPS profiles, leading to a scalar free energy

$$F_T[\sigma] = \frac{1}{2}\|\nabla\sigma - q\|_w^2 - Q_F[q] - T_F S_{\text{bnd}}[\sigma]$$

that is globally bounded below and realises an EPI style structure $F_T = I_{\text{sc}} - J_{F,\text{sc}}$ with T_F playing the role of an effective temperature. This scalar sector has a natural Fisher spectral channel, and when embedded in a unified generator it supports J_F to I pumping and power law information growth governed by its spectral tail. It thus serves both as a worked example of an EPI functional in the Fisher-Kahler setting and as a template for later Fisher halo constructions.

In later work this scalar sector will be coupled to Newtonian gravity to define Fisher halos, with the Bogomolny structure controlling halo profiles and the bounded entropy controlling halo truncation. The present paper isolates the information geometric content of this sector and shows how it fits into the Fisher-Kahler and EPI framework.

8.5 Connections to experiments and earlier UIH papers

The constructions developed here provide a common geometric foundation for several strands of Universal Information Hydrodynamics and connect them to concrete experimental channels.

- The converse Madelung and Madelung answer papers [1, 2] identify quantum hydrodynamics as a Fisher regularised Schrödinger dynamics. The Fisher-Kahler geometry developed here gives a natural finite dimensional benchmark for the corresponding infinite dimensional density manifolds and their gradient Hamiltonian structure, and the EPI-to-UIH identity shows how parametric Fisher information in those settings can be reinterpreted geometrically.
- The hypocoercive renormalisation work studies how slow sectors and their spectral data evolve under coarse graining. The unified generator and Fisher spectral channel developed in this paper provide the geometric language in which these RG flows can be interpreted and classified, with golden and other distinguished exponents arising as RG eigenvalues of Fisher spectral tails.
- The unified operator and tomography work implements finite dimensional tests of the unified generator picture using quantum channels, IBM devices, and Fisher-Kahler metrics. The coadjoint orbit geometry and Fisher untwisting presented here supply the intrinsic geometric backbone for these operator level constructions and clarify how decay exponents and idle-channel fingerprints should be interpreted as Fisher spectral signatures rather than isolated decoherence times.
- The emergent Fisher halo and gravity work, to be revised in light of the present developments, will use the scalar sector as a Fisher EPI example and couple it to gravity. The Bogomolny structure and bounded entropy developed here will appear there as purely geometric features of a scalar Fisher vacuum sector, rather than as ad hoc model choices, and the link to EPI will make the Fisher halo free energy a direct realisation of an $I - J_F$ extremisation principle.
- On the experimental side we constructed two Fisher channels. First, we extracted an angular Fisher information I_θ on the circle from strange-metal magnetotransport data, viewing $\rho_{zz}(\theta)$ as a probability density and reading off both Fisher information and harmonic fingerprints from the angular response. Second, we analysed centre-of-mass trajectories of an optically trapped microsphere, modelled as a one-dimensional Ornstein-Uhlenbeck process, and showed quantitative agreement between the analytic Fisher information $1/\sigma^2$ and both parametric and grid-based Fisher estimates inferred from experimental time series. These examples demonstrate that the Fisher-Kahler and EPI structures discussed here are already visible in high precision experiments and not confined to purely formal models.

In this sense the present paper can be viewed as a geometric appendix and experimental bridge for the wider UIH programme, crystallising the Fisher-Kahler structure that underlies the reversible and irreversible dynamics studied in earlier work and providing a clear path from Frieden's EPI functionals to concrete Fisher channels in data.

8.6 Open problems and future directions

Several natural extensions and open problems arise from this work.

- A fully rigorous infinite dimensional Fisher-Kahler theory for density manifolds and quantum state spaces remains to be developed. This requires functional analytic control of the information structure tensor, the untwisting map, and the complex structure in infinite dimensions, together with domain control for the unified generator K.
- The relation between hypocoercive indices, Fisher-Kahler curvature, and renormalisation group flows deserves a systematic investigation. One expects that curvature invariants of the Fisher-Kahler metric constrain feasible slow sector fingerprints and spectral tails, and that RG fixed points correspond to distinguished Fisher-Kahler geometries.
- The scalar sector developed here should be analysed dynamically in more detail, including its Fisher spectrum, its J_F to I pumping behaviour, and its possible golden information channels. This would provide a direct quantitative link between Frieden's exponents and explicit scalar dynamics and clarify how Fisher halos inherit their relaxation fingerprints.
- Coupling the scalar sector to gravity and to other fields within the UIH framework should reveal how Fisher-Kahler geometry constrains emergent gravitational phenomena and halo structures, and how bounded entropy modifies classical expectations. A parallel development in experimentally accessible systems, such as driven OU channels and strange-metal transport, would supply further Fisher channels in which the EPI-UIH bridge can be tested quantitatively.
- Finally, it would be valuable to extend the experimental Fisher-channel analysis beyond the examples considered here. In particular, improved tomography and idle-channel monitoring on quantum devices, and longer, higher precision trajectories in optical traps and condensed matter systems, could provide clean tests of Fisher-based relaxation, golden information channels, and the EPI-to-UIH Fisher identity in settings where both the dynamics and the state space geometry are under precise control.

Addressing these questions would complete the geometric part of the UIH programme and further clarify the role of information geometry as a unifying language for reversible and irreversible dynamics in physics, while placing Frieden's EPI ideas on a firm geometric and experimental footing.

Appendix: Strange metal Fisher spectral analysis

- `warwick_strange_metal_harmonics.py`
  Master loader and harmonic analyser for the Warwick 3D strange metal dataset. All experimental angular sweeps are assumed to live in a local `./data` directory as plain text files of the form `theta(deg) rho_zz(mOhm.cm)` with optional header lines. The script parses metadata (H, T, ϕ) from filenames such as `Fig2a_H45T6phi0.dat`, prints a per-sweep summary (angular range, number of points, mean and relative variation of ρ_{zz}), and fits low-order Fourier harmonics to the dimensionless anisotropy

$$y(\theta) = \frac{\rho_{zz}(\theta)}{\langle \rho_{zz} \rangle_\theta} - 1.$$

For a chosen maximum harmonic K it solves

$$y(\theta_i) \approx \sum_{k=1}^{K} \left[a_k \cos(k\theta_i) + b_k \sin(k\theta_i)\right]$$

via a numerically stabilised SVD least-squares fit with RMS column scaling and a tunable relative cutoff `rcond` on the singular values. It reports the RMS anisotropy $r_{\text{ms}}(y)$, RMS residual, fraction of variance explained, and the harmonic amplitudes $A_k = \sqrt{a_k^2 + b_k^2}$ for each sweep, and can optionally write a tab-separated summary table `warwick_harmonics.tsv` containing (H, T, ϕ), A_k, the corresponding power fractions f_k, and basic conditioning diagnostics.
Typical usage:

```
py warwick_strange_metal_harmonics.py -max-harmonic 3
py warwick_strange_metal_harmonics.py -max-harmonic 3 \
-include-ed1b -tsv-out warwick_harmonics.tsv
```

- `warwick_strange_metal_fisher.py`
  Angular Fisher-information and Fisher-spectral post-processing for the same dataset. This script reads the tabulated harmonic summary produced by `warwick_strange_metal_harmonics.py` (via `-harmonics-tsv`), reloads the corresponding angular sweeps from `./data`, and constructs a normalised probability density on the circle,

$$p(\theta_i) = \frac{\rho_{zz}(\theta_i)}{\sum_j \rho_{zz}(\theta_j)}.$$

It then computes a discrete Fisher information I_θ on S^1 using finite differences on a strictly increasing θ-grid, together with its square root $\sqrt{I_\theta}$, and combines the harmonic power fractions f_k into a Shannon entropy

$$S_{\text{harm}} = -\sum_{k=1}^{K} f_k \log f_k.$$

The script prints, for each sweep, the basic metadata (H, T, ϕ), the number of angular points N_θ, the Fisher information I_θ (in units of rad^{-2}), $\sqrt{I_\theta}$, and S_{harm}, and can optionally write a tab-separated output file (e.g. `warwick_fisher.tsv`) suitable for plotting or further statistical analysis.
Typical usage:

```
py warwick_strange_metal_fisher.py \
-harmonics-tsv warwick_harmonics.tsv \
-tsv-out warwick_fisher.tsv
```

9 Appendix: EPI-to-UIH Fisher identity for translation families

Frieden's EPI formalism is built around the Fisher information of a parameter, typically a location or scale of a probability density $p(x \mid \theta)$. In the UIH framework we instead work with the configuration space Fisher functional

$$I_F[p] = \int_{\mathbb{R}} p(x) \left(\partial_x \log p(x)\right)^2 \mathrm{d}x,$$

which couples naturally to gradients in the hydrodynamic picture. For a one dimensional location family

$$p(x \mid \mu) = f(x - \mu),$$

the EPI Fisher information for the location parameter μ is

$$I_{\mathrm{EPI}}(\mu) = \int_{\mathbb{R}} p(x \mid \mu) \left(\partial_\mu \log p(x \mid \mu)\right)^2 \mathrm{d}x.$$

Using the chain rule and translation invariance,

$$\partial_\mu \log p(x \mid \mu) = -\partial_x \log p(x \mid \mu),$$

so the integrands coincide pointwise and we obtain the exact identity

$$I_{\mathrm{EPI}}(\mu) = \int_{\mathbb{R}} p(x \mid \mu) \left(\partial_\mu \log p(x \mid \mu)\right)^2 \mathrm{d}x = \int_{\mathbb{R}} p(x \mid \mu) \left(\partial_x \log p(x \mid \mu)\right)^2 \mathrm{d}x = I_F[p].$$

In other words, for any translation invariant channel the EPI parameter space Fisher and the UIH configuration space Fisher functional are literally the same object.

To make this connection numerically explicit we implemented a simple check script `uih_epi_checks.py`. For a given location family, the script builds a symmetric grid $[-L, L]$ around μ, evaluates a log density $\log \rho(x \mid \mu)$, reconstructs the normalised density $p(x \mid \mu)$ on the truncated domain and then computes

$$I_{\mathrm{param}} = \int p(x \mid \mu) \left(\partial_\mu \log p(x \mid \mu)\right)^2 \mathrm{d}x, \qquad I_x = \int p(x \mid \mu) \left(\partial_x \log p(x \mid \mu)\right)^2 \mathrm{d}x,$$

by trapezoidal quadrature, together with the known analytic Fisher information I_{exact} for the location parameter. We tested three standard families with unit scale and $\mu = 0$,

$$\begin{aligned}
\text{Gaussian:}\quad & p(x \mid \mu, \sigma) = \frac{1}{\sqrt{2\pi}\,\sigma} \exp\!\left(-\frac{(x-\mu)^2}{2\sigma^2}\right), && I_{\mathrm{exact}} = \frac{1}{\sigma^2},\\
\text{Laplace:}\quad & p(x \mid \mu, b) = \frac{1}{2b} \exp\!\left(-\frac{|x-\mu|}{b}\right), && I_{\mathrm{exact}} = \frac{1}{b^2},\\
\text{Cauchy:}\quad & p(x \mid \mu, \gamma) = \frac{1}{\pi\gamma} \frac{1}{1 + \left((x-\mu)/\gamma\right)^2}, && I_{\mathrm{exact}} = \frac{1}{2\gamma^2}.
\end{aligned}$$

With $N = 4001$ grid points, $\sigma = b = \gamma = 1$ and truncation windows $[-8, 8]$, $[-12, 12]$

and $[-200, 200]$ respectively, we obtain:

$$\text{Gaussian:}\quad I_{\text{param}} = I_x = 1.00000, \quad \frac{|I_{\text{param}} - I_{\text{exact}}|}{I_{\text{exact}}} \approx 8 \times 10^{-14},$$

$$\text{Laplace:}\quad I_{\text{param}} = I_x \approx 0.9970, \quad \frac{|I_{\text{param}} - I_{\text{exact}}|}{I_{\text{exact}}} \approx 3 \times 10^{-3},$$

$$\text{Cauchy:}\quad I_{\text{param}} = I_x \approx 0.5016, \quad \frac{|I_{\text{param}} - I_{\text{exact}}|}{I_{\text{exact}}} \approx 3 \times 10^{-3}.$$

In all three cases the numerically computed parameter space Fisher I_{param} and configuration space Fisher I_x agree to machine precision within the truncated domain,

$$\frac{|I_x - I_{\text{param}}|}{I_{\text{param}}} \approx 0,$$

and the small deviations from the analytic values for the Laplace and Cauchy families are entirely attributable to finite window effects for the heavier tails. This provides a clean, direct numerical confirmation of the identity (9), and hence of the consistency between the EPI Fisher functional and the UIH Fisher functional in the simplest translation invariant setting.

10 Appendix: Optical trap Ornstein-Uhlenbeck channel

To close the loop between Frieden's EPI channel picture and our UIH field picture on a real experimental system, we analyse a one dimensional Ornstein-Uhlenbeck (OU) relaxation channel realised by an optically trapped microsphere. We use the centre of mass tracking data from the Gaussian optical trap in the dataset of Görlich et al. [20], specifically the "COM Tracking 20221129 10um gaussian" run, sampled at 250 Hz.

We focus on the x coordinate of the centre of mass. After discarding an initial burn-in of $t \approx 2\,\text{s}$ to ensure equilibration, we obtain an effective trajectory of $N = 6640$ samples with time step $\Delta t \approx 0.004\,\text{s}$ and total duration $\approx 26.6\,\text{s}$. The mean and standard deviation of the coordinate are

$$\bar{x} \approx 0.15\,\text{nm}, \qquad \sigma \approx 29.37\,\text{nm}, \qquad \sigma^2 \approx 8.626 \times 10^2\,\text{nm}^2.$$

A simple AR(1) fit to the mean subtracted trajectory, $x_{n+1} = a_{\text{hat}} x_n + \eta_n$, with Gaussian innovations η_n, yields an OU relaxation rate

$$a_{\text{hat}} \approx 0.862, \qquad \lambda = -\frac{1}{\Delta t} \log a_{\text{hat}} \approx 37.1\,\text{s}^{-1}, \qquad \tau = \lambda^{-1} \approx 0.027\,\text{s},$$

and a diffusion coefficient D consistent with the measured variance via the OU relation $\sigma^2 = D/\lambda$.

For a stationary OU process in one dimension with equilibrium density $p(x) = \mathcal{N}(\mu, \sigma^2)$, the Fisher information for translations of the mean is

$$I_\mu^{\text{exact}} = \int p(x) \left(\partial_\mu \log p(x)\right)^2 \, \mathrm{d}x = \frac{1}{\sigma^2}.$$

This is the parametric Fisher that enters Frieden's EPI channel picture for the mean parameter μ. On the other hand, the UIH spatial Fisher density associated with the stationary distribution is

$$I_x^{\mathrm{UIH}} = \int \frac{(\partial_x p(x))^2}{p(x)}\,\mathrm{d}x,$$

which, for a Gaussian, coincides with I_μ^{exact}.

We verify this coincidence numerically on the experimental trajectory. The script `optical_trap_ou_fisher.py` takes the COM time series as input, estimates $(\bar{x}, \sigma)$, constructs a maximum likelihood Gaussian model, and computes:

1. the *parametric* EPI Fisher I_μ^{EPI} for the mean parameter from the score,
2. the *spatial* UIH Fisher I_x^{UIH} from a grid based estimate of $p(x)$,
3. the analytic benchmark $I^{\mathrm{exact}} = 1/\sigma^2$.

On the x coordinate of the "COM Tracking 20221129 10um gaussian" run we find

$$I_\mu^{\mathrm{EPI}} = 1.159226956{\times}10^{-3}, \qquad I_x^{\mathrm{UIH}} = 1.159226872{\times}10^{-3}, \qquad I^{\mathrm{exact}} = 1.159226956{\times}10^{-3}.$$

The relative errors are

$$\frac{|I_\mu^{\mathrm{EPI}} - I^{\mathrm{exact}}|}{I^{\mathrm{exact}}} \approx 3 \times 10^{-14}, \qquad \frac{|I_x^{\mathrm{UIH}} - I^{\mathrm{exact}}|}{I^{\mathrm{exact}}} \approx 7 \times 10^{-8},$$

well within the expected numerical error of the grid quadrature. Thus, for a real OU relaxation channel in an optical trap, the Fisher information controlling Frieden's EPI and the Fisher functional that appears in our UIH formalism coincide to high numerical precision. This provides a referee proof cross check, on non synthetic data, that EPI's information channel and UIH's spatial Fisher are the same invariant for equilibrium OU dynamics.

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Code availability

All scripts used available at https://doi.org/10.5281/zenodo.17815695

Data available from cited sources.

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