Geometric Bogomolny Bounds

Universal Information Hydrodynamics: Bogomolny bounds, curvature, and equality certificates

J. R. Dunkley

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Context. In the historical SUSY story, "BPS protection" is algebraic: a mass–charge bound follows from a central extension, and saturation is tied to multiplet shortening. UIH does not assume SUSY, strings, or any related structures. It recovers a closely related *mathematical core*: an exact Bogomolny square completion gives a sharp bound with a first-order equality condition. UIH formulates an operational analogue of SUSY-style "protection" in terms of (i) a computable defect-to-bound functional, (ii) curvature-driven relaxation to the minimiser, and (iii) equality diagnostics that certify when the system is actually on the predicted channel.

One sentence definition. In UIH the corresponding protection mechanism is *geometric*: a Bogomolny lower bound fixes a sharp floor, information curvature fixes a relaxation rate, and equality certificates (BPS defect, equality dial, no-work identity) make the claim falsifiable in numerics and data.

1. The Bogomolny completion is the primitive

Let a scalar Fisher channel be described by a field σ on X, with weight $w(x) > 0$ and a source sector encoded by $q(x)$. The cold (zero Fisher temperature) pure-sector free energy admits an *exact* square completion

$$F_0[\sigma] \;=\; \frac{1}{2}\int_X w\,|\nabla\sigma - q|^2\,d\mu \;-\; Q_F[q], \qquad Q_F[q] := \frac{1}{2}\int_X w\,|q|^2\,d\mu.$$

This identity gives the sharp bound

$$F_0[\sigma] \geq -Q_F[q],$$

with equality if and only if $\nabla\sigma = q$ a.e. (in the stated function class and boundary conditions). We use "BPS" only in this restricted Bogomolny sense: *saturating the completed-square bound*, with no supersymmetry assumed.

2. The operational dial is the BPS defect

Define the *defect-to-bound* functional

$$\Delta_{\text{BPS}}[\sigma; q] \;:=\; F_0[\sigma] + Q_F[q] \;=\; \frac{1}{2}\int_X w\,|\nabla\sigma - q|^2\,d\mu \;\geq\; 0.$$

This plays the role of a "multiplet protection" diagnostic: it is a nonnegative, computable number. Small Δ_{BPS} means close-to-cold saturation; large Δ_{BPS} signals a deformed regime (finite Fisher temperature, stiffness running, boundary flux, or coupling to additional sectors).

The generic minimiser solves the weighted Euler–Lagrange condition

$$\nabla\cdot\big(w(\nabla\sigma - q)\big) = 0,$$

and the pointwise BPS equation $\nabla\sigma = q$ is the sharp equality subclass (attained when q is compatible with being a gradient in the relevant class).

3. Protection by curvature: why saturation becomes an attractor

A bound alone is static. UIH adds a second ingredient: **positive information curvature** (curvature coercivity of the Fisher functional in the natural H_ρ^{-1} geometry). When curvature has a uniform lower floor $\kappa_{\min} > 0$, Fisher gradient flows inherit quantitative relaxation estimates of the form

$$F(t) - F_\infty \;\leq\; \big(F(0) - F_\infty\big)\,e^{-2\kappa_{\min} t},$$

so the minimiser is not merely a special configuration but a dynamically selected attractor, with a measurable rate. This is the UIH analogue of "radiative stability": perturbations decay because the geometry is coercive.

4. Equality certificates: how we test the mechanism

UIH supplies three clean certificates that mirror some roles played by SUSY protection arguments, but framed without superalgebra.

1. **BPS defect:** Δ_{BPS} is the exact nonnegative gap to the Bogomolny floor. BPS saturation is equivalent to $\Delta_{\text{BPS}} = 0$.
2. **Equality dial (irreversible sector):** the sharp cost–entropy inequality has equality *iff* the protocol tangent is collinear with the Onsager gradient-flow direction. This certifies whether the observed dynamics is genuinely the Fisher gradient flow, rather than merely fit by it.
3. **No-work identity (reversible sector):** the antisymmetric Liouville channel does no work on the free energy. Reversible circulation can change trajectories, but it cannot change the dissipation budget required to reach a given free-energy level.

Together, these certificates separate (i) being near the BPS bound, (ii) being on the true gradient-flow ray, and (iii) being contaminated by reversible circulation.

5. Relation to SUSY-style BPS protection

The historical SUSY BPS story uses symmetry algebra to argue robustness: a central extension fixes a mass–charge bound, and multiplet shortening explains why small perturbations do not spoil saturation. UIH realises a closely analogous pattern using *inequality, geometry, and diagnostics* rather than superalgebra. In this restricted, operational sense the same structural ingredients appear:

- **Bounded below:** the Bogomolny completion fixes a hard floor determined by the source sector.
- **Rigidity:** saturation selects a first-order equation (a sharp structural regime).
- **Attractor:** positive curvature forces relaxation toward the minimiser at a measurable rate.
- **Certificates:** defect, equality dial, and no-work identity indicate which regime the system is in.

Thus, for macroscopic information-theoretic models, SUSY is not required in order to obtain BPS-like robustness: the protected structure (and its controlled failure modes) already follows from the Fisher geometry and its associated inequalities.