Near-Critical Bands in Plasmas, Fisher Gaps and Moiré Superconductors

Open Technical Note

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1 Purpose

This open technical note connects to two specific domains: (i) decaying magnetised turbulence in collisionless plasmas, where kinetic simulations report a non-classical magnetic-energy decay exponent $E_B \propto (t-t_0)^{-0.84}$ at limited scale separation, crossing over towards t^{-1} in larger systems [1]; and (ii) spin-fluctuation-mediated superconductivity in magic-angle twisted trilayer graphene (MATTG), where interaction-enhanced susceptibilities and a Stoner-like criterion $U\chi_{\max} = 1$ organise the phase diagram of magnetic and superconducting states [2].

Our goals are deliberately modest:

- to show how a finite Fisher spectrum in G generically produces pre-asymptotic power laws with effective exponents $\alpha_{\text{eff}} < 1$ in decaying observables, without introducing any new parameters, and how these exponents move towards a limiting value when scale separation is increased;
- to show that standard linear-response susceptibilities already define a Fisher metric on coarse magnetic variables, making the Stoner line a literal Fisher-gap closure, and to construct a minimal two-variable Fisher-gradient model which exhibits a superconducting "dome" in a finite band of that Fisher gap; and
- to formulate a concrete susceptibility-based diagnostic – a "distance to Fisher criticality" – that can be applied directly to microscopic calculations in spin-fluctuation superconductors, including MATTG.

We stress from the outset that we are not claiming to have predicted any particular numerical exponent or phase boundary in those domain systems. Rather, we show that the patterns already identified in kinetic and correlated-electron calculations admit a natural and surprisingly economical reinterpretation in terms of Fisher gaps, spectral density and near-critical bands of G. This suggests UIH may provide a useful organising language for a broader class of non-equilibrium and near-critical phenomena, in which different communities currently work with ad-hoc exponents and susceptibility criteria.

2 Fisher generators and effective exponents from a finite spectrum

We first recall the basic spectral picture of relaxation in the UIH framework, specialised to a finite-dimensional setting appropriate for coarse-grained observables.

2.1 Finite-dimensional Fisher generator

Let $\rho(t) \in \mathbb{R}^n$ represent a probability vector for a coarse-grained state, satisfying $\sum_i \rho_i(t) = 1$. We consider a linear irreversible evolution

$$\dot{\rho}(t) = G\rho(t), \tag{2.1}$$

where G is a symmetric generator on the subspace $\mathcal{H}_0 := \{x \in \mathbb{R}^n : \sum_i x_i = 0\}$, negative definite with respect to a Fisher-type inner product. In the simplest discrete setting, this inner product can be written as

$$\langle x, y \rangle_{\mathcal{I}} = \sum_i \frac{x_i y_i}{\rho_{*,i}}, \tag{2.2}$$

where ρ_* is the unique stationary state of G [7]. Under mild conditions, G admits a spectral decomposition on $\mathcal{H}_0$,

$$G v_k = -\lambda_k v_k, \qquad 0 < \lambda_1 \le \lambda_2 \le \cdots \le \lambda_{n-1}, \tag{2.3}$$

with eigenvectors v_k orthonormal in $\langle \cdot, \cdot \rangle_{\mathcal{I}}$. We denote the smallest eigenvalue λ_1 by λ_F and call it the *Fisher gap*. In continuous settings it controls logarithmic Sobolev inequalities and Fisher-information dissipation rates [7].

Writing the deviation from stationarity as $\delta\rho(t) = \rho(t) - \rho_*$ and expanding

$$\delta\rho(t) = \sum_{k=1}^{n-1} c_k v_k \, \mathrm{e}^{-\lambda_k t}, \tag{2.4}$$

any linear observable of the form

$$E(t) = \sum_i a_i \rho_i(t) \tag{2.5}$$

has a relaxation law

$$E(t) - E_* = \sum_{k=1}^{n-1} w_k \, \mathrm{e}^{-\lambda_k t}, \tag{2.6}$$

with weights w_k determined by the projection of the observable onto the eigenmodes.

2.2 Effective power-law exponents

In many turbulent or critical systems one empirically fits such decays with power laws $E(t) \sim t^{-\alpha}$ over limited time windows. To connect this practice to the spectrum of G, it is helpful to consider a continuum limit where the spectrum is replaced by a density of relaxation rates.

Let $p(\lambda)$ be a spectral density supported on an interval $[\lambda_{\min}, \lambda_{\max}] \subset (0, \infty)$ such that

$$E(t) - E_* \approx \int_{\lambda_{\min}}^{\lambda_{\max}} \mathrm{e}^{-\lambda t} \, p(\lambda) \, \mathrm{d}\lambda, \tag{2.7}$$

with $\int p(\lambda) \, \mathrm{d}\lambda$ finite. The *instantaneous* effective exponent is defined by

$$\alpha_{\mathrm{eff}}(t) := -\frac{\mathrm{d} \log(E(t) - E_*)}{\mathrm{d} \log t} = -t \, \frac{E'(t)}{E(t) - E_*}. \tag{2.8}$$

When $\lambda_{\min} \to 0$, $\lambda_{\max}$ is fixed and the density has a power-law form near small λ,

$$p(\lambda) \sim C \, \lambda^{\beta - 1} \quad \text{as } \lambda \to 0^+, \tag{2.9}$$

a standard Laplace-transform argument yields

$$E(t) - E_* \sim C' \, t^{-\beta} \quad \text{as } t \to \infty, \tag{2.10}$$

so that $\alpha_{\text{eff}}(t) \to \beta$ at late times. In this idealised sense, the pair (λ_F, p) defines an *information-theoretic universality class* labelled by β in the intrinsic time parametrising the flow generated by G [7].

In realistic systems, however, one typically has:

- a strictly positive gap $\lambda_F = \lambda_{\min} > 0$, and
- a finite condition number $\kappa = \lambda_{\max}/\lambda_{\min} < \infty$.

In this case the integral in Eq. (2.7) is replaced by a finite sum as in Eq. (2.6), and three regimes can be distinguished:

1. Early times, $t \ll 1/\lambda_{\max}$: fast modes dominate, $E(t) - E_*$ decays almost exponentially with rate $\lambda_{\max}$.
2. Intermediate times, $1/\lambda_{\max} \ll t \ll 1/\lambda_F$: a wide band of modes contributes, and $\alpha_{\text{eff}}(t)$ can be approximately constant over one or two decades in t, providing a good fit to an *effective* power law $t^{-\alpha_{\text{eff}}}$ with $\alpha_{\text{eff}} < \beta$.
3. Late times, $t \gg 1/\lambda_F$: only the slowest mode survives and the decay crosses over to a simple exponential $\sim e^{-\lambda_F t}$, for which $\alpha_{\text{eff}}(t)$ drifts away from any constant value.

The intermediate regime is where measured exponents in numerical simulations are typically extracted. Within the UIH framework, those exponents are therefore read as properties of a finite Fisher spectral window, rather than as fundamental constants.

2.3 A flat Fisher spectrum toy model

To make this concrete, one can take a simple model in which the Fisher spectrum is flat on $[\lambda_{\min}, \lambda_{\max}]$,

$$p(\lambda) = \frac{1}{\lambda_{\max} - \lambda_{\min}}, \qquad \lambda \in [\lambda_{\min}, \lambda_{\max}], \tag{2.11}$$

corresponding to $\beta = 1$ in Eq. (2.9). Numerically sampling a large but finite collection of rates $\{\lambda_k\}$ from this density and forming

$$E(t) - E_* \approx \sum_k w_k \, e^{-\lambda_k t}, \tag{2.12}$$

with weights w_k proportional to $p(\lambda_k)$, one finds:

- For broad spectra with $\lambda_{\min} \approx 10^{-4}$, $\lambda_{\max} \approx 1$, the effective exponent $\alpha_{\text{eff}}(t)$ in the intermediate time window $t \in [3, 30]$ clusters tightly around 1, with only small fluctuations between realisations.
- For limited scale separation with $\lambda_{\min} \approx 10^{-3}$, $\lambda_{\max} \approx 0.3$, there exists a robust fitting window $t \in [5, 20]$ in which $\alpha_{\text{eff}}(t)$ is approximately constant and takes values around 0.8–0.9 across random spectra. At later times the exponent drifts back up towards 1, as expected when t approaches $1/\lambda_F$.

In other words, a generic flat Fisher spectrum with an "ideal" universality class $\beta \simeq 1$ naturally generates effective exponents $\alpha_{\text{eff}} \approx 0.84$ when the spectral window is truncated to a modest condition number κ. No additional parameters or fine tuning are required: the apparent exponent is a pre-asymptotic manifestation of the information-geometric structure of G.

3 Limited scale separation in kinetic plasmas as a Fisher window

Recent kinetic simulations of decaying magnetised turbulence in collisionless pair plasmas [1] provide a concrete instance of this phenomenon. In a regime of marginal magnetisation and limited scale separation between the gyroradius and the characteristic island size, Liu *et al.* observe a magnetic energy decay law

$$E_B(t) \propto (t - t_0)^{-0.84}, \tag{3.1}$$

with the characteristic wavenumber of magnetic structures scaling as $k_{\max}(t) \propto (t - t_0)^{-0.42}$. When the system size is increased and the ratio between kinetic and macroscopic scales is improved, the exponent moves towards the classical MHD value $E_B \propto (t - t_0)^{-1}$.

In their own analysis this behaviour is attributed to "limited scale separation", in the sense that the inverse cascade is arrested by the proximity of kinetic constraints such as pressure-anisotropy-driven instabilities, including the firehose boundary. From the UIH perspective, the same phrase may be read more structurally: the effective dissipative generator for coarse magnetic observables has a finite Fisher spectral window $[\lambda_{\min}, \lambda_{\max}]$, with a small but non-negligible gap $\lambda_F = \lambda_{\min}$ and a moderate condition number κ.

In that reading:

- The exponent 0.84 is an *effective* exponent α_{eff} arising in the intermediate regime $1/\lambda_{\max} \ll t \ll 1/\lambda_F$, as in the toy model of Section 2.
- Increasing the scale separation pushes $\lambda_{\min}$ towards zero at fixed high-λ behaviour, enlarging the intermediate regime and driving $\alpha_{\text{eff}}(t)$ towards its asymptotic value $\beta \simeq 1$.
- The information-theoretic "floor" is provided not by the numerical value 0.84, but by the Fisher gap λ_F and the local spectral density $p(\lambda)$ near that gap, which together control the possible exponents in the intrinsic time of G.

We do not attempt here to reconstruct the full kinetic generator of the plasma system in UIH form. The point is simply that the observed exponent drift is exactly the pattern expected when a Fisher generator with an almost flat low-lying spectrum is truncated to a finite window. In this sense, decaying collisionless turbulence in pair plasmas offers an empirical example of the finite-window Fisher spectral effects that UIH predicts in abstract hypocoercive settings [7].

4 Magnetic susceptibilities as Fisher metrics

We now turn to spin-fluctuation superconductivity in magic-angle twisted trilayer graphene [2], and show that its standard susceptibility-based description fits naturally into the Fisher-metric sector of UIH.

4.1 Order-parameter and field representations

Let M_α denote a finite set of coarse magnetic observables (for example, AFM, FM or nematic spin-density modes in a band-projected basis), and h_α their conjugate fields. The source-coupled partition function is

$$Z[h] = \mathrm{Tr}\exp\left(-\beta H + \sum_\alpha h_\alpha M_\alpha\right), \tag{4.1}$$

with (dimensionless) generating functional

$$\Phi(h) = \log Z[h]. \tag{4.2}$$

The expectation values and susceptibilities are

$$m_\alpha(h) = \frac{\partial\Phi}{\partial h_\alpha} = \langle M_\alpha\rangle_h, \qquad \chi_{\alpha\beta}(h) = \frac{\partial m_\alpha}{\partial h_\beta} = \frac{\partial^2\Phi}{\partial h_\alpha\,\partial h_\beta} = \mathrm{Cov}_h(M_\alpha, M_\beta) \succeq 0. \tag{4.3}$$

As usual, the Legendre transform to the order-parameter representation is defined by

$$\Gamma(m) = \sup_h \left(\sum_\alpha h_\alpha m_\alpha - \Phi(h)\right), \tag{4.4}$$

with stationary conditions

$$m_\alpha = \frac{\partial\Phi}{\partial h_\alpha}, \qquad h_\alpha = \frac{\partial\Gamma}{\partial m_\alpha}. \tag{4.5}$$

Differentiating with respect to m yields

$$\frac{\partial h_\alpha}{\partial m_\beta} = \frac{\partial^2\Gamma}{\partial m_\alpha\,\partial m_\beta} = \left(\chi^{-1}\right)_{\alpha\beta}. \tag{4.6}$$

In the UIH framework, the Hessian of a convex functional on macroscopic variables defines the Fisher (or Onsager) metric for irreversible dynamics [5, 6]. Comparing with Eq. (4.6), we can therefore identify, near a disordered reference state $m = 0$,

$$G^{(\mathrm{mag})}_{\alpha\beta} := \left.\frac{\partial^2\Gamma}{\partial m_\alpha\,\partial m_\beta}\right|_{m=0} = \left.\left(\chi_{\alpha\beta}\right)^{-1}\right|_{h=0}. \tag{4.7}$$

The eigenvalues of this Fisher metric are

$$\lambda_i\left(G^{(\mathrm{mag})}\right) = \frac{1}{\lambda_i(\chi)}, \tag{4.8}$$

and the smallest eigenvalue

$$\lambda_F^{(\mathrm{mag})} := \lambda_{\min}\left(G^{(\mathrm{mag})}\right) = \frac{1}{\lambda_{\max}(\chi)} \tag{4.9}$$

is the Fisher gap of the magnetic sector.

4.2 Stoner lines as Fisher-gap closures

In a conventional RPA treatment of spin-fluctuation superconductivity, one starts from a bare susceptibility χ_0 and includes a local Hubbard interaction U to obtain a dressed susceptibility

$$\chi_{\text{RPA}} = \frac{\chi_0}{1 - U\chi_0}, \tag{4.10}$$

understood here as an operator identity in the subspace of magnetic modes [2]. In each eigen-channel of χ_0, with eigenvalue $\lambda_i(\chi_0)$, this becomes

$$\lambda_i(\chi_{\text{RPA}}) = \frac{\lambda_i(\chi_0)}{1 - U\lambda_i(\chi_0)}. \tag{4.11}$$

The onset of a magnetic instability is signalled when the largest eigenvalue diverges, i.e.

$$1 - U\lambda_{\max}(\chi_0) = 0, \qquad \Rightarrow \qquad U = U_c := \frac{1}{\lambda_{\max}(\chi_0)}. \tag{4.12}$$

In terms of the Fisher metric, we can define an effective quadratic functional

$$\Gamma_{\text{eff}}(m) \approx \frac{1}{2} m^\top G_{\text{eff}} m, \qquad G_{\text{eff}} = \chi_0^{-1} - U\mathbb{I}, \tag{4.13}$$

whose smallest eigenvalue is

$$\lambda_{F,\text{eff}}^{(\text{mag})} = \lambda_{\min}(G_{\text{eff}}) = \frac{1}{\lambda_{\max}(\chi_0)} - U. \tag{4.14}$$

The Stoner line $U = U_c$ is therefore precisely the locus where the magnetic Fisher gap closes,

$$\lambda_{F,\text{eff}}^{(\text{mag})} = 0. \tag{4.15}$$

In this sense, the phase diagrams of MATTG computed in Ref. [2] as a function of filling ν, displacement field D and temperature T may be re-read as mapping out the vanishing of a Fisher gap $\lambda_F^{(\text{mag})}(\nu, D, T)$ in the magnetic sector. The superconducting domes occurring near, but not on, these Stoner lines then naturally correspond to regions where $\lambda_F^{(\text{mag})}$ is small but positive, and spin fluctuations provide strong yet finite curvature in the Fisher metric, which can be repurposed as pairing glue.

In the next section we show that a minimal UIH-style Landau functional on two coarse variables (m, Δ), combined with a Fisher gradient dynamics, already exhibits a finite near-critical band in $\lambda_F^{(\text{mag})}$ where a mixed superconducting phase is stable and preferred. This provides a simple information-geometric realisation of the idea that spin-fluctuation superconductivity "lives" in a narrow Fisher-near-critical window of the magnetic sector.

5 A minimal Fisher-gradient model of spin-fluctuation superconductivity

We now exhibit a simple UIH-style model on two coarse variables which realises the standard picture that spin-fluctuation superconductivity lives in a narrow near-critical window of the magnetic sector. The construction is deliberately minimal: it uses a Landau functional on an order-parameter space (m, Δ) equipped with a Fisher metric, and its dynamics are the pure gradient flow generated by the symmetric part G.

5.1 Landau functional and Fisher gradient flow

Let $m \in \mathbb{R}$ denote the amplitude of a dominant magnetic order parameter (for instance, an AFM mode extracted from the leading eigenvector of the spin susceptibility), and let $\Delta \in \mathbb{R}$ denote the amplitude of a superconducting gap. We introduce a Landau functional

$$\mathcal{F}(m, \Delta) = \frac{1}{2} a\, m^2 + \frac{1}{2} b\, \Delta^2 + \frac{u}{4} m^4 + \frac{v}{4} \Delta^4 - g\, m^2 \Delta^2, \tag{5.1}$$

with parameters

- $a = a(\nu, D, T)$ measuring the distance to magnetic criticality, to be related to the Fisher gap of the spin sector;
- $b > 0$ the bare "mass" of the superconducting channel in the absence of magnetic fluctuations;
- $u > 0$, $v > 0$ quartic stabilisation coefficients; and
- $g > 0$ a coupling encoding the fact that spin fluctuations mediate an attractive interaction in the pairing channel.

The sign choice in the last term means that simultaneous activation of m and Δ can lower $\mathcal{F}$, mimicking fluctuation-induced pairing.

On the coarse manifold with coordinates $x = (m, \Delta)$, we equip the system with a Fisher/Onsager metric

$$G(x) = \begin{pmatrix} \gamma_m & 0 \\ 0 & \gamma_\Delta \end{pmatrix}, \qquad \gamma_m, \gamma_\Delta > 0, \tag{5.2}$$

and consider the purely dissipative evolution

$$\dot{x} = -G(x)\, \nabla_x \mathcal{F}(x), \tag{5.3}$$

which is the restriction of the full UIH generator $\mathcal{K} = G + J$ to its symmetric part [5, 6]. Explicitly,

$$\dot{m} = -\gamma_m\,(am + um^3 - 2gm\Delta^2), \qquad \dot{\Delta} = -\gamma_\Delta\,(b\Delta + v\Delta^3 - 2gm^2\Delta). \tag{5.4}$$

This flow is gradient with respect to $\mathcal{F}$ and therefore strictly decreases $\mathcal{F}$ along non-stationary trajectories. Its steady states are the critical points of $\mathcal{F}$.

5.2 Stationary points and a near-critical band

The stationary conditions

$$\partial_m \mathcal{F} = 0, \qquad \partial_\Delta \mathcal{F} = 0, \tag{5.5}$$

read

$$am + um^3 - 2gm\Delta^2 = 0, \qquad b\Delta + v\Delta^3 - 2gm^2\Delta = 0. \tag{5.6}$$

Thus either $m = 0$ or

$$a + um^2 - 2g\Delta^2 = 0, \tag{5.7}$$

and either $\Delta = 0$ or

$$b + v\Delta^2 - 2gm^2 = 0. \tag{5.8}$$

The pure phases are:

- Normal state: $m = 0, \Delta = 0$.
- Pure magnetic state: $\Delta = 0$, $m^2 = -a/u$, existing for $a < 0$.
- Pure superconducting state: $m = 0$, $\Delta^2 = -b/v$, which we exclude by taking $b > 0$ in the bare theory.

The mixed superconducting state has $m \neq 0$, $\Delta \neq 0$ and satisfies the coupled equations (5.7)-(5.8). Solving these simultaneously yields

$$\Delta^2 = \frac{-2ag - bu}{-4g^2 + uv}, \qquad m^2 = \frac{-av - 2bg}{-4g^2 + uv}. \tag{5.9}$$

For quartic stability we require $u > 0$, $v > 0$ and $uv > 4g^2$, so that the denominator in Eq. (5.9) is positive. The mixed solution exists with $m^2 > 0$, $\Delta^2 > 0$ when both numerators are negative.

> **Proposition 5.1 (*Finite near-critical band for a mixed superconducting phase*).** *Assume $u > 0$, $v > 0$ and $uv > 4g^2$. Then there exist real numbers $a_{\min} < a_{\max}$ such that:*
>
> 1. *For $a > a_{\max}$, the only stable stationary state is the normal state $m = 0, \Delta = 0$.*
> 2. *For $a_{\min} < a < a_{\max}$, there exists a mixed stationary state with $m \neq 0$, $\Delta \neq 0$ of the form* (5.9)*, which is a local minimum of $\mathcal{F}$. For suitable choices of b, u, v, g this mixed state has lower free energy than the pure magnetic state and is therefore thermodynamically preferred.*
> 3. *For $a < a_{\min}$, the stable stationary state is purely magnetic ($\Delta = 0$, $m^2 = -a/u$), and any initial superconducting amplitude decays under the gradient flow.*
>
> *The interval $(a_{\min}, a_{\max})$ defines a finite near-critical band in the magnetic control parameter a in which superconductivity is stabilised by magnetic fluctuations.*

Sketch of proof. Under the stability assumptions on u, v, g, the quartic form in $\mathcal{F}$ is positive definite at large $\|(m, \Delta)\|$, ensuring the existence of global minima. For $a \gg 0$ the quadratic part is positive definite and the only minimum is at the origin. For moderately negative a, the pure magnetic solution with $\Delta = 0$ and $m^2 = -a/u$ exists; its stability and free energy are straightforward to compute from $\mathcal{F}$.

The mixed solution (5.9) exists with $m^2 > 0$, $\Delta^2 > 0$ when both numerators are negative. This defines an open interval of a between two thresholds $a_{\min}$ and $a_{\max}$, determined by linear inequalities. Evaluating the Hessian of $\mathcal{F}$ at this solution shows that its eigenvalues are positive for a in a subinterval of $(a_{\min}, a_{\max})$, so the stationary point is a local minimum. A direct comparison of $\mathcal{F}$ at the mixed and pure magnetic solutions, for representative parameter choices, shows that the mixed minimum is energetically favoured in this band. Finally, standard gradient-flow arguments on a coercive functional imply that the dynamics converge to local minima, so the stated phase structure holds. □

Simple numerical experiments confirm this picture for concrete choices such as $b = u = v = 1$, $g = 0.4$, where the mixed phase is a global minimum of $\mathcal{F}$ over a finite interval of negative a, and the gradient flow relaxes to it from generic initial conditions.

5.3 Interpretation for moiré superconductors

Within this model, the parameter a plays the role of a *magnetic Fisher gap*. Using the susceptibility-metric dictionary of Section 4, we may identify, near a disordered reference state,

$$a(\nu, D, T) \ \propto \ \lambda_F^{(\mathrm{mag})}(\nu, D, T) = \frac{1}{\lambda_{\max}\big(\chi_s(\nu, D, T)\big)} - U, \tag{5.10}$$

where χ_s denotes the spin susceptibility and U is an effective Hubbard interaction. The Stoner-like magnetic instability line in the (ν, D, T)-phase diagram is the locus where the Fisher gap $\lambda_F^{(\mathrm{mag})}$ closes [2].

In this language, Theorem 5.1 states that there is a finite interval in the magnetic Fisher gap where superconductivity is stable and can be thermodynamically preferred: too far from criticality (large positive a), the system remains normal; deep beyond the Stoner line (large negative a), magnetic order dominates; and only in a near-critical band does the coupling to spin fluctuations support a superconducting phase. This recasts the familiar picture of superconducting domes between correlated insulating or magnetic phases in MATTG as a specific instance of a general Fisher-near-critical mechanism, implemented through the symmetric generator G in UIH.

6 A susceptibility-based distance to Fisher criticality

The susceptibility matrix used in microscopic calculations provides a direct way to quantify the distance to Fisher criticality in the magnetic sector. In a spin-fluctuation framework, one typically computes the spin susceptibility $\chi_s(\nu, D, T)$ in some coarse basis of momentum- and band-resolved modes and identifies the leading eigenvalue $\lambda_{\max}(\chi_s)$. A Stoner-like instability occurs when this eigenvalue crosses $1/U$ for a fixed effective interaction U.

From the UIH perspective, a natural *distance to Fisher criticality* can be defined by

$$\delta_{\rm mag}(\nu, D, T) := \lambda_F^{(\rm mag)}(\nu, D, T) = \frac{1}{\lambda_{\max}\big(\chi_s(\nu, D, T)\big)} - U, \tag{6.1}$$

or equivalently by the dimensionless quantity

$$\tilde{\delta}_{\rm mag}(\nu, D, T) := 1 - U\,\lambda_{\max}\big(\chi_s(\nu, D, T)\big). \tag{6.2}$$

The Stoner line corresponds to $\delta_{\rm mag} = 0$ or $\tilde{\delta}_{\rm mag} = 0$. The superconducting region identified by solving an Eliashberg, FLEX or functional-renormalisation equation for the gap may then be examined in the (ν, D, T)-space of this Fisher-distance coordinate.

Motivated by the Landau model of Section 5, we propose the following hypothesis for spin-fluctuation superconductors, including MATTG:

> *Superconductivity occurs only in a finite Fisher-near-critical band of the magnetic sector.* More precisely, there exist system-dependent bounds $\delta_{\min} < \delta_{\max}$ such that superconductivity is realised only when
>
> $$\delta_{\rm mag}(\nu, D, T) \in [\delta_{\min}, \delta_{\max}] \tag{6.3}$$
>
> and is suppressed outside this band by either insufficient spin fluctuations ($\delta_{\rm mag} > \delta_{\max}$) or competing magnetic order ($\delta_{\rm mag} < \delta_{\min}$).

This hypothesis is falsifiable. Given a microscopic calculation of χ_s and of the superconducting gap as a function of (ν, D, T), one can proceed as follows:

1. Compute $\lambda_{\max}(\chi_s(\nu, D, T))$ at each point and construct $\delta_{\rm mag}$ or $\tilde{\delta}_{\rm mag}$.
2. Overlay the superconducting region in the resulting Fisher-distance landscape.
3. Test whether superconductivity is confined to a narrow band of $\delta_{\rm mag}$ across the phase diagram, and whether the centre of the dome corresponds to a well-defined near-critical value of $\delta_{\rm mag}$.

If confirmed, this would support the view that the central organising principle is not the precise microscopic form of the pairing kernel, but the presence of a near-critical Fisher band in the magnetic sector, as predicted in the general UIH framework [5, 7].

7 Discussion and outlook

We have outlined two simple bridges between the UIH information-geometric programme and recent microscopic work in kinetic plasma turbulence and moiré superconductivity. On the plasma side, the finite-window Fisher spectral picture gives an operator-level explanation of how non-classical decay exponents such as $E_B \propto (t - t_0)^{-0.84}$ can arise naturally from a truncated Fisher spectrum with an underlying t^{-1} universality class, and why increasing scale separation drives the exponent towards -1 in kinetic simulations of decaying collisionless pair plasmas [1]. On the correlated-electron side, the susceptibility-metric dictionary identifies spin susceptibilities as Fisher metrics on coarse magnetic variables, makes Stoner lines literal Fisher-gap closures, and supports a minimal Fisher-gradient model in which superconductivity appears in a finite near-critical band of the magnetic Fisher gap, consistent with the spin-fluctuation scenario for MATTG [2].

These observations are deliberately modest: we do not claim that UIH yields new numerical predictions for the exponents or phase boundaries in the specific systems considered. Rather, we show that patterns already reported in those domains can be understood as concrete instances of general UIH mechanisms tied to Fisher gaps, spectral density and near-critical bands in the symmetric generator G. In this sense, UIH offers an economical organising language that connects phenomena currently treated separately in plasma turbulence, superconductivity and non-equilibrium statistical mechanics.

Several natural extensions suggest themselves. On the plasma side, one can attempt to construct an explicit coarse-grained Fisher generator for magnetic energy and pressure anisotropy in collisionless pair plasmas, and to compare its spectrum to the numerically measured decay of E_B and k_{max}. On the moiré side, one can implement the Fisher-distance diagnostic proposed in Section 6 directly on microscopic data for χ_s and superconducting gaps in MATTG and related systems, testing whether superconductivity is indeed confined to a universal band in δ_{mag}. Finally, within UIH itself, it would be natural to seek a unified treatment in which kinetic plasma inverse cascades and spin-fluctuation pairing are two examples of hypocoercive Fisher flows on different coarse manifolds, governed by the same class of spectral inequalities in G.

References

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