# Recovering General Relativity from Fisher Kinematics

Universal Information Hydrodynamics: refoliation invariance forces Einstein geometrodynamics

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16th December 2025

**Context: why this matters.** The weak field phenomenology in the gravity draft adopts the Einstein-Hilbert term as the minimal covariant completion of the scalar Fisher sector. Here is a stronger and cleaner statement: once the kinematics on the space of spatial metrics are fixed by the Fisher geometry, demanding **genuine refoliation invariance** (no preferred slicing) essentially forces the reversible gravitational backbone to be **Einstein geometrodynamics**.

> **One sentence theorem:** Adopt a Fisher information metric on the configuration space of spatial geometries (a DeWitt-type supermetric) and require the reversible generator to represent the hypersurface deformation (Dirac) constraint algebra; then the Hamiltonian constraint is fixed to the ADM form of General Relativity (with Newton's constant and $\Lambda$ as free couplings, plus matter terms).

## 1. Setup: Fisher geometry on the space of 3-metrics

Consider a spatial slice $\Sigma$ with Riemannian metric $g_{ij}$. Let $h_{ij}$ be a symmetric perturbation of $g_{ij}$, and define the trace $h := g^{ij}h_{ij}$. A Fisher information construction on the space of spatial metrics produces a DeWitt-type quadratic form on perturbations:

$$G(h,h) \;=\; \alpha \int_{\Sigma} \left( h_{ij}h^{ij} + \beta\, h^2 \right) \sqrt{g}\, d^3x, \qquad \alpha > 0, \tag{1}$$

for constants $\alpha$ and $\beta$. Up to constants, this is the algebraic structure of the DeWitt supermetric used in canonical gravity.

**UIH reading.** Equation (1) is the kinematical Fisher metric on configuration space (superspace). It is not an assumption about dynamics; it is a statement about the information geometry controlling how metric configurations are distinguished by the theory's state space structure.

- **Kinematics:** Fisher geometry gives a quadratic form on $h_{ij}$ of DeWitt type.
- **Phase space:** introduce conjugate momenta $\pi^{ij}$, so the reversible sector lives on $(g_{ij}, \pi^{ij})$.
- **Target symmetry:** relativistic dynamics means the theory cannot depend on a preferred foliation of spacetime into slices.

## 2. Refoliation invariance as a constraint algebra

Refoliation invariance is encoded by requiring the reversible generator on $(g_{ij}, \pi^{ij})$ to represent the hypersurface deformation (Dirac) algebra of constraints.

Denote by $D[\vec{N}]$ the diffeomorphism constraint (smeared with shift vector field $\vec{N}$) and by $H[N]$ the Hamiltonian constraint (smeared with lapse function $N$). Refoliation invariance requires the Poisson brackets to close as

$$\{D[\vec{N}_1], D[\vec{N}_2]\} = D[[\vec{N}_1, \vec{N}_2]], \tag{2}$$

$$\{H[N], D[\vec{N}]\} = H[\mathcal{L}_{\vec{N}} N], \tag{3}$$

$$\{H[N_1], H[N_2]\} = D\Big[g^{ij}(N_1\partial_j N_2 - N_2\partial_j N_1)\Big]\,. \tag{4}$$

The crucial point is (4): the bracket of two Hamiltonian constraints closes into a diffeomorphism constraint with **structure functions** involving the spatial metric $g^{ij}$. That is the algebraic signature of refoliation invariance, and it is much stronger than mere spatial diffeomorphism invariance.

> **Interpretation:** The reversible generator must implement the same deformation algebra as spacetime hypersurfaces. If it fails, the dynamics selects a preferred slicing and is not genuinely relativistic.

## 3. Uniqueness: the ADM constraint is forced

Given a DeWitt-type supermetric in the kinetic term (fixed kinematics) and the requirement that (2) to (4) hold (fixed symmetry), the form of the Hamiltonian constraint is essentially fixed.

One obtains the ADM Hamiltonian constraint of Einstein gravity (with cosmological constant), plus matter:

$$H \;=\; \frac{1}{\sqrt{g}}\left(\pi_{ij}\pi^{ij} - \frac{1}{2}\pi^2\right) \;-\; \sqrt{g}\,(R - 2\Lambda) \;+\; H_{\text{matter}} \;\approx\; 0, \tag{5}$$

together with the momentum constraint

$$H_i \;=\; -2\nabla_j \pi^j{}_i \;+\; H_i^{\text{matter}} \;\approx\; 0. \tag{6}$$

Here $R$ is the scalar curvature of $g_{ij}$ and $\pi := g_{ij}\pi^{ij}$. The coefficient $1/2$ in (5) corresponds to the general-relativistic value of the DeWitt supermetric parameter: it is the refoliation-invariant choice singled out by constraint closure.

**Covariant completion.** Legendre transforming the canonical system (5) and (6) yields the covariant Einstein-Hilbert action. Newton's constant and $\Lambda$ remain as free couplings, as expected.

**What is forced and what is not:** The *form* of the reversible gravitational backbone is fixed by kinematics plus refoliation invariance, but the overall coupling (Newton's constant) and $\Lambda$ are not fixed by this argument. Matter contributions enter through $H_{\text{matter}}$ and $H_i^{\text{matter}}$.

## 4. UIH reading: where GR sits inside the larger structure

Within UIH, the generator splits into a symmetric part (dissipative, Fisher) and an antisymmetric part (reversible, Hamiltonian). This note is only about the reversible backbone.

- **Reversible sector:** once you insist on refoliation invariance, the antisymmetric generator on $(g_{ij}, \pi^{ij})$ must implement the hypersurface deformation algebra, and this forces the ADM form.
- **Scalar Fisher sector:** the Fisher halo degrees of freedom and their screening introduce additional effective stress-energy contributions in the weak field, but they do not replace the Einstein backbone. They sit on top of it as a structured vacuum response sector.
- **Currents:** any additional reversible current degrees of freedom must either be absorbed into an extended constraint system that still closes, or be suppressed (for example by isotropy or screening regimes) in the phenomenological solver limit.

## 5. Practical consequences for the gravity draft

This result is valuable because it cleanly separates two layers:

1. **Backbone:** the reversible geometrodynamics is GR when refoliation invariance holds.
2. **Phenomenology:** deviations in observables (slip, lensing modifications, effective density loading, screening features) come from the additional Fisher response sector and its couplings, not from abandoning the Einstein constraint structure.

So when speaking to a sceptic: you are not proposing a replacement for GR. You are proposing a specific structured vacuum response sector coupled to GR, with a reversible limit that is forced back to Einstein geometrodynamics by symmetry.

## 6. Minimal falsifiable programme for this note

Even though this is a structural argument, it has operational hooks:

- **Constraint closure diagnostics:** in any discretisation or solver implementation of the reversible sector, measure the failure of (2) to (4) under numerical Poisson brackets. Constraint algebra drift is a quantitative measure of foliation artefacts.
- **Supermetric parameter check:** if you attempt a more general DeWitt-type family (varying the trace weight), the refoliation closure requirement singles out the GR value corresponding to the $1/2$ in (5). This can be implemented as a one-parameter sweep with an algebra residual.
- **Separation of roles:** once the reversible backbone is fixed, any mismatch to data is forced to live in the Fisher response sector. This prevents parameter creep: you cannot hide a bad phenomenology inside an arbitrary modification of the Hamiltonian constraint.

**Takeaway:** Fisher geometry fixes the kinematics on superspace. Refoliation invariance fixes the reversible dynamics. Together they lock the backbone to Einstein geometrodynamics, leaving UIH-specific phenomenology to appear where it should: in the structured vacuum response sector (screening, slip, lensing, and environment dependence).

## References (minimal)


- Any standard canonical GR text for ADM and the Dirac constraint algebra.
- The uniqueness logic (constraint algebra closure forcing the ADM form) is the classical geometrodynamics result often attributed to the hypersurface deformation algebra programme.