Local Complexifier Rigidity

Universal Information Hydrodynamics: no hidden local change of variables

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Context: why "complexifying" matters. In the reversible sector we work in hydrodynamic variables (ρ, S) with continuity $\partial_t \rho + \nabla \cdot (\rho \nabla S/m) = 0$ and a Hamilton–Jacobi equation containing a local curvature term. A standard objection to any hydrodynamic-first approach is: "perhaps your apparent uniqueness is an artefact of variables, and some other local reparametrisation makes a different reversible theory linear too". Local complexifier rigidity closes that door inside the admissible class.

One sentence theorem (Prop. 8.1). If a local, pointwise, invertible, gauge-covariant complexifier

$$\psi = F(\rho)\, e^{iG(S,\rho)}, \qquad F > 0,$$

maps the Fisher-regularised reversible hydrodynamics into a *linear* Schrödinger evolution

$$i\kappa\, \partial_t \psi = \Big(-\frac{\kappa^2}{2m}\Delta + V \Big)\psi \quad \text{with the same external } V(x) \text{ and constant } \kappa > 0,$$

then, up to constant phase and scale,

$$F(\rho) = c\sqrt{\rho}, \qquad G(S,\rho) = \frac{S}{\kappa} + \text{const}, \qquad \alpha = \frac{\kappa^2}{2m}.$$

Equivalently, $\psi = \sqrt{\rho}\, e^{iS/\kappa}$ is the only admissible local linearising map (up to scale), and it fixes the Fisher scale.

1. What is assumed (and what is not)

Locality. $\psi(\rho, S)$ is zeroth order in spatial derivatives of (ρ, S). **Gauge covariance.** Global $U(1)$ on S is encoded by $S \mapsto S + \sigma$, acting as a global phase on ψ, so $G(S+\sigma, \rho) - G(S, \rho)$ is independent of x. **Linearity.** The target PDE is linear with coefficients independent of (ρ, S), meaning external potentials only. **Invertibility.** $F > 0$ and $G_S \neq 0$ almost everywhere on the positivity set.

This does *not* classify derivative-dependent or nonlocal complexifications. Those raise differential order and exit the class by construction.

2. Why the conclusion is forced (the proof mechanism)

Two structural identities do all the work.

(i) Flux matching fixes the phase gradient and the amplitude. Linear Schrödinger evolution implies the continuity law $\partial_t |\psi|^2 + \nabla \cdot J = 0$ with

$$J = \frac{\kappa}{m}\, \mathrm{Im}(\bar{\psi}\nabla\psi) = \frac{\kappa}{m} F(\rho)^2 \nabla G.$$

Hydrodynamics gives $\partial_t \rho + \nabla \cdot (\rho \nabla S/m) = 0$, so the current is $j = \rho \nabla S/m$. Gauge covariance forces G to be affine in S: requiring $\psi \mapsto e^{i\sigma/\kappa}\psi$ under $S \mapsto S + \sigma$ gives $G(S+\sigma, \rho) - G(S, \rho) = \sigma/\kappa$, hence $G_S \equiv 1/\kappa$. Therefore

$$\nabla G = \frac{1}{\kappa}\nabla S + G_\rho(\rho)\, \nabla\rho, \qquad J = \frac{1}{m} F(\rho)^2 \nabla S + \frac{\kappa}{m} F(\rho)^2 G_\rho\, \nabla\rho.$$

Matching $J \equiv j$ for arbitrary admissible states forces the $\nabla\rho$ coefficient to vanish and the ∇S coefficient to match:

$$G_\rho \equiv 0, \qquad F(\rho)^2 \propto \rho,$$

so $F(\rho) = c\sqrt{\rho}$ with $c > 0$.

(ii) Recombination fixes the Fisher scale. With the polar map $\psi = \sqrt{\rho}\, e^{iS/\kappa}$ fixed (up to c and constant phase), the hydrodynamic system recombines into a linear Schrödinger equation if and only if the curvature coefficient matches

$$\alpha = \frac{\kappa^2}{2m}.$$

Setting $\kappa = \hbar$ yields $\alpha = \hbar^2/(2m)$.

3. What this buys you

No hidden local linearisation. Within the admissible class there is no second local change of variables that could secretly linearise a different reversible theory while keeping the axioms and exact projective linearity intact.

A single scale. The same local complex structure fixes the Fisher curvature scale. In particular the free-particle dispersion is pinned once κ is identified operationally.

A clean refutation criterion. A counterexample *within scope* must either identify a concrete error in the derivations under the axioms, or exhibit an explicit model satisfying all axioms (including coefficient-only linearity) while producing a non-Schrödinger reversible flow. Models outside axiomatic scope do not refute the classification, they redefine the problem.

4. Node handling (the only subtle analytic point)

All equalities are meant on the positivity set $\{\rho > 0\}$ and extend in the weak sense using test functions, with the quotient $\Delta R/R$ interpreted distributionally (where $R = \sqrt{\rho}$). This is consistent with the admissible boundary classes and the functional-analytic setup used elsewhere in the programme.