The Metric Adjoint Split

Universal Information Hydrodynamics: separating dissipation from circulation

J. R. Dunkley

December 2025

Context. In UIH the generator is not “Hamiltonian vs dissipative” by assumption. We start with a single effective evolution operator K acting on an information state space, and then *extract* the irreversible and reversible channels from geometry. The extraction is canonical: it depends only on the choice of inner product induced by the Fisher–Kähler / BKM metric at a reference state.

One sentence definition. Given a positive definite metric M (Fisher/BKM at ρ_{ss}), the *metric adjoint split* writes any generator K uniquely as $K = G + J$ where G is the symmetric (dissipative) part and J is the skew (circulating) part in the M-geometry.

1. The construction (canonical once M is fixed)

Work on a complex coordinate space with the M-inner product

$$\langle u, v\rangle_M := u^\dagger M v, \qquad M \succ 0.$$

The **metric adjoint** of K is the unique operator $K^\sharp$ satisfying $\langle u, Kv\rangle_M = \langle K^\sharp u, v\rangle_M$ for all u, v, namely

$$K^\sharp = M^{-1} K^\dagger M.$$

Then define

$$G := \tfrac{1}{2}\big(K + K^\sharp\big), \qquad J := \tfrac{1}{2}\big(K - K^\sharp\big), \qquad \Rightarrow \qquad K = G + J.$$

By construction,

$$MG = (MG)^\dagger, \qquad MJ = -(MJ)^\dagger,$$

so G is M-self-adjoint and J is M-skew-adjoint. (If working in a real coordinate representation, replace $\dagger$ by transpose.)

2. What it means physically (no-work vs production)

Let the quadratic information functional be

$$F(u) = \tfrac{1}{2}\, u^\dagger M u, \qquad \dot{u} = Ku.$$

Then

$$\dot{F} = \mathrm{Re}\big(u^\dagger M K u\big) = \mathrm{Re}\big(u^\dagger M G u\big), \qquad \mathrm{Re}\big(u^\dagger M J u\big) = 0.$$

So J performs **no work**: it redistributes without changing F. All production (entropy growth, information decay, cost) is carried by G. This is the precise meaning of “irreversible vs reversible” in UIH.

3. Why this is not a choice (uniqueness)

Once the metric M is fixed, the split is unique. Any other decomposition $K = \tilde{G} + \tilde{J}$ with $M\tilde{G}$ Hermitian and $M\tilde{J}$ skew-Hermitian must coincide with the above, because

$$\tilde{G} = \tfrac{1}{2}(K + K^\sharp) = G, \qquad \tilde{J} = \tfrac{1}{2}(K - K^\sharp) = J.$$

So “reversible vs irreversible” is not assigned by hand. It is read off from geometry.

4. Two concrete consequences used in the programme

(i) Density-sector universality. In the GKLS setting, place the BKM metric at a full-rank stationary state ρ_{ss}. Then $G = \frac{1}{2}(K + K^\sharp)$ restricted to diagonal perturbations reproduces the classical Fisher Dirichlet dissipation of the induced Markov density dynamics. Hamiltonian dressing changes J, but leaves the diagonal Dirichlet form invariant.

(ii) Operational diagnostics. Given observed probe responses, one can estimate the Gram matrix $B_{ij} = \langle \varphi_i, G\varphi_j\rangle_M$ and test whether the inferred generator admits a consistent M-skew part J. Failures show up as violations of $MG = (MG)^\dagger$ or nonzero $\mathrm{Re}(u^\dagger M J u)$ on test vectors.

5. Minimal falsifiers (what would actually break the claim)

1. **Metric inconsistency:** show that the claimed M is not the correct Fisher/BKM metric at ρ_{ss}, or that it fails to be positive definite on the claimed state space. Then the split is being taken in the wrong geometry.
2. **Adjoint identity failure:** exhibit u, v such that $\langle u, Kv\rangle_M \neq \langle K^\sharp u, v\rangle_M$ for $K^\sharp = M^{-1}K^\dagger M$. That would be an algebraic contradiction (it will not happen if $M \succ 0$ and the representation is consistent).
3. **No-work violation:** find a quadratic $F(u) = \frac{1}{2}u^\dagger M u$ with $\dot{u} = Ku$ where $\mathrm{Re}(u^\dagger M J u) \neq 0$ despite MJ being skew-Hermitian. That would contradict linear algebra.

6. Practical takeaway

The metric adjoint split is the UIH “wiring diagram” for dynamics: it converts any generator into a symmetric G (irreversible power) and a skew J (circulation), using only the Fisher/BKM geometry. This is the mechanism that lets one unify Schrödinger, Fokker–Planck, Markov, and GKLS evolution under one operator $K = G + J$ without hand-labeling what is “Hamiltonian” and what is “dissipative”.