Projective Superposition Test

Universal Information Hydrodynamics: operational witness of exact projective linearity

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Context. In the reversible sector, the admissible completion is singled out by an exact complex structure on rays. A concise operational falsifier is to test whether the evolved ray of a sum equals the sum of evolved rays, up to a global phase. This is the "projective" version of superposition.

Definition (projective residual). Pick two initial states $\psi_1(0), \psi_2(0)$. Let

$$\psi_\oplus(t) := U(t)\,\frac{\psi_1(0)+\psi_2(0)}{\sqrt{2}}, \qquad \psi_\Sigma(t) := \frac{U(t)\psi_1(0)+U(t)\psi_2(0)}{\sqrt{2}}.$$

Define the ray distance by normalising and optimally aligning the global phase,

$$R_{\mathrm{proj}}(t) := \min_{\theta\in[0,2\pi)} \left\| \frac{\psi_\oplus(t)}{\|\psi_\oplus(t)\|_2} - e^{i\theta}\,\frac{\psi_\Sigma(t)}{\|\psi_\Sigma(t)\|_2} \right\|_2.$$

Pass condition. Exact projective linearity implies $R_{\mathrm{proj}}(t)=0$ for all t, up to numerical tolerance. **Claim in scope.** In the admissible local first order reversible class, the Fisher regulariser is the unique choice that achieves this once the unique local complexifier is imposed.

1. What to probe

For a genuinely linear Schrödinger evolution $U(t)$, superposition holds on vectors, hence also on rays, so $R_{\mathrm{proj}}(t)$ collapses to numerical floor. Any state dependent term breaks this even if it is real and norm preserving, because $|\psi_1+\psi_2|^2$ contains cross terms that are not present when evolving ψ_1 and ψ_2 separately.

2. Setup

Two displaced Gaussian packets are prepared,

$$\psi_{1,2}(x,0) = \frac{1}{(\pi\sigma^2)^{1/4}}\exp\Big(-\frac{(x-x_{1,2})^2}{2\sigma^2}\Big)\,\exp\Big(\frac{i}{\hbar}p_{1,2}(x-x_{1,2})\Big),$$

and evolved in a harmonic trap $V(x)=\frac{1}{2}m\omega^2x^2$, chosen to force overlap so that interference is unavoidable. For the Fisher choice, the flow is linear Schrödinger. To model a non Fisher local curvature in the hydrodynamic energy, a small local positive proxy is added which becomes a real, state dependent potential in the ψ-picture,

$$i\hbar\,\partial_t\psi = \Big(-\frac{\hbar^2}{2m}\Delta + V(x)\Big)\psi + U_\beta[\rho]\psi, \qquad \rho=|\psi|^2, \qquad U_\beta[\rho]=\beta\,\frac{|\nabla\rho|^2}{(\rho+\varepsilon)^2}.$$

Evolution is performed by Strang split step Fourier, with a single refinement run (grid $N\to 2N$, timestep $dt\to dt/2$) to separate physics from discretisation.

3. Typical Results

Representative residuals $R_{\mathrm{proj}}(T)$ (base vs refined).

Model	Base grid	Refined grid
Linear ($\beta=0$)	6.2×10^{-14}	1.2×10^{-13}
Nonlinear ($\beta=0.005$)	1.65×10^{-1}	3.08×10^{-1}
Nonlinear ($\beta=0.01$)	8.41×10^{-1}	1.41
Nonlinear ($\beta=0.02$)	1.40	1.41
Nonlinear ($\beta=0.05$)	1.41	1.41

Interpretation. The Fisher linear case sits at numerical floor and stays there under refinement. Any non Fisher perturbation yields a finite residual that does not vanish under refinement or phase alignment.

4. Why this matters in the programme

This is a cheap, sharp witness for "exact projective linearity" in the sense used elsewhere as a consistency criterion for a Hilbert space realisation. It closes a common escape route in discussions: it is not enough that a candidate curvature looks small, or that some subset of wave packets behaves approximately linearly. Inside the stated admissible class, either the dynamics is exactly linear on rays (Fisher corner), or superposition fails and the theory is operationally distinct.