Universal Information Hydrodynamics

A common generator for Markov, Fokker Planck and GKLS flows

J. R. Dunkley

21st November 2025

Abstract

We work in the universal information hydrodynamics (UIH) framework developed in two companion papers, in which reversible quantum dynamics and irreversible response are organised in a local metriplectic geometry driven by a convex free energy, a Fisher information metric, and an antisymmetric no-work channel. Within this setting the irreversible drift is always a Fisher gradient flow in a weighted H^{-1} geometry, equipped with a cost entropy inequality and a curvature coercivity bound that are invariant under the addition of compatible reversible dynamics. In the present paper we show that this structure has a canonical realisation for Fokker Planck equations, finite Markov chains and GKLS semigroups, and that it can be reconstructed operationally from process and state tomography on real quantum hardware.

On the classical side we identify, for any reversible Markov generator Q with stationary law π, a Fisher Dirichlet operator $G = Q\,\mathrm{diag}(\pi)$ that saturates the cost entropy inequality mode by mode and generates a Fisher gradient flow of the relative entropy in the hydrodynamic Fokker Planck limit. On the quantum side we show that, in a wide class of thermal, nonreversible and coherent GKLS models, the symmetric part of the real generator in the stationary BKM metric has a density block that coincides exactly with this classical Fisher operator, so that population dynamics are always classical Fisher information hydrodynamics while circulation and coherence effects live in the skew part J. We prove a finite dimensional UIH hypocoercivity theorem in which the large time decay rate of the semigroup generated by $K = G + J$ is bounded below by a positive multiple of the Fisher spectral gap of G, and we construct a simple renormalisation scheme that preserves Fisher dissipation on chosen observables and exhibits a robust diffusive Fisher universality basin. In one dimensional Fisher diffusions this scheme supports a Fisher analogue of the Jarzynski relation with renormalisation stable free energy differences.

We implement a suite of IBM Quantum experiments that realise this picture on a superconducting qubit. Process tomography of idle and driven circuits yields an effective real generator K in a Pauli basis whose metric adjoint split in the BKM metric at the device stationary state produces a clean metriplectic decomposition $K = G + J$; independent idle depths are consistent with a single time homogeneous generator; the dissipative spectrum of G sets an information theoretic speed limit for BKM Fisher decay; and a curvature test shows that the same BKM metric is the local Hessian of quantum relative entropy at the stationary state. On both synthetic models and hardware reconstructions the BKM metric, the Hamiltonian symplectic form and a compatible complex structure assemble into a Kähler triple that is approximately preserved by UIH coarse graining, while cross coherence diagnostics turn the reconstructed generators into a practical UIH universality spectrometer. All claims are backed by a reproducible Python archive that includes constructive Fokker Planck to Markov to GKLS lifts, hypocoercive decay diagnostics, fluctuation and renormalisation experiments, and hardware K tomography. Taken together, these results provide evidence that the UIH programme is both mathematically coherent and experimentally visible, with emerging signatures of universality in gaps, fluctuation free energies and Kähler RG structure.

Contents

1 Introduction

Open quantum systems are most often described either at the microscopic level by GKLS master equations for density matrices or at the macroscopic level by irreversible transport equations and stochastic differential equations. Between these two descriptions lies a growing body of work in which dissipative evolution is organised as a gradient flow in an information geometry, with Fisher metrics, entropy production and hypocoercive decay replacing bare operator norms as the natural objects. In practice, however, the reversible and irreversible channels of a given open-system model are rarely assembled into a single geometric structure, and it is not clear a priori which parts of that structure are universal and which depend on microscopic details.

In two companion papers [1, 2] we introduced a universal information hydrodynamics (UIH) framework that begins to address this question. The reversible channel is written as a Hamiltonian flow on a pair (ρ, S) of density and phase variables, while the irreversible channel is generated by a symmetric mobility operator G that determines both a Fisher information metric and a weighted H^{-1} geometry. A single convex free energy drives the irreversible drift, and the combination of G with an antisymmetric no-work operator J produces a local metriplectic structure. Within this setting we proved a cost entropy inequality that links control cost to entropy production, and a curvature coercivity bound that links Fisher curvature to the irreversible drift. Both statements are invariant under the addition of any reversible flow generated by an antisymmetric operator satisfying a weighted Liouville condition.

The first aim of the present paper is to show that this abstract structure has a canonical realisation in both the Fokker–Planck and reversible Markov settings, and a numerically rigid realisation across a wide class of GKLS semigroups. On the classical side, once a reversible Markov generator Q and its stationary distribution π are fixed, there is a unique Fisher–Dirichlet operator

$$G = Q\,\mathrm{diag}(\pi)$$

that saturates the cost entropy inequality mode by mode and realises the irreversible channel as a Fisher gradient flow. In the hydrodynamic limit this operator generates an overdamped Fokker–Planck equation with the same free energy, Fisher metric and Dirichlet structure, placing discrete reversible chains and continuum Fokker–Planck equations on a single UIH ladder.

On the quantum side we show that for a broad class of GKLS models, including coherent and nonreversible examples, the density block of the metric-symmetric part G of the real generator in the stationary BKM metric is exactly this same classical Fisher operator. Population dynamics are therefore always classical Fisher information hydrodynamics, with all coherence and circulation effects living in the antisymmetric part J. We construct explicit lifts from continuum Fokker–Planck flows to reversible Markov chains and diagonal GKLS generators, and show that coherent Hamiltonian dressing leaves the density-sector Fisher geometry unchanged. Many distinct GKLS models therefore share the same irreversible Fisher hydrodynamics, differing only in their skew channels.

The second aim is to study the large-time behaviour of the full UIH generator $K = G+J$. We prove a finite dimensional UIH hypocoercivity theorem in which the asymptotic decay rate of the semigroup generated by K is bounded below by a positive multiple of the Fisher spectral gap of G, with the proportionality constant depending only on dimensionless coupling parameters that measure the strength of J relative to G. Once the stationary geometry and Fisher gap are fixed, coherent circulation can reshape trajectories and induce non-normal transients but cannot arbitrarily slow the irreversible drift along density modes. We then introduce a UIH renormalisation group that coarse-grains generators while preserving Fisher dissipation on chosen observables, and we show numerically that a wide class of Fokker–Planck, Markov and GKLS models flow to a diffusive Fisher universality basin. In one-dimensional Fisher diffusions this provides a Fisher analogue of the Jarzynski relation with renormalisation-stable free energy differences.

The third aim is to demonstrate that this picture can be reconstructed operationally. Using IBM Quantum hardware we perform process and state tomography of idle and driven circuits on a superconducting qubit, reconstruct effective short-time generators K via matrix logarithms, and obtain clean metriplectic decompositions $K = G + J$ in the stationary BKM metric. Independent idle depths agree with a single time-homogeneous generator. The dissipative spectrum of G sets an information-theoretic speed limit for the decay of the BKM quadratic functional, and a curvature diagnostic confirms that the same BKM metric is the local Hessian of quantum relative entropy. A complex-structure test shows that the BKM metric, Hamiltonian symplectic form and a compatible complex structure organise into an approximate Kähler triple that is preserved by Fisher-preserving coarse-graining. Cross coherence diagnostics turn the reconstructed generators into a practical UIH universality spectrometer, distinguishing full UIH closures from diagonal baselines and identifying Fisher-active modes.

All results are supported by a reproducible Python archive that includes constructive Fokker–Planck to Markov to GKLS lifts, hypocoercive decay diagnostics, renormalisation experiments, Fisher–Jarzynski and Kähler RG tests, IBM K-tomography, curvature and BKM speed-limit diagnostics, and permanent links to the code in Appendix A.

1.1 Reader roadmap

Section 2 recalls reversible Fisher hydrodynamics and the metriplectic framework, fixing notation for the Fisher metric, reversible current and entropy production. It emphasises that the same Fisher geometry underlies both the reversible Madelung picture and the irreversible metriplectic structure.

Section 3 introduces finite dimensional information manifolds and shows how a UIH generator K on perturbations decomposes as $K = G + J$ into a Fisher-gradient part G and a Hamiltonian part J. It defines the metric, the Fisher gap on the traceless subspace, and the basic hypocoercive setting used in the rest of the paper.

Section 4 constructs diagonal GKLS generators from detailed-balance Markov chains and identifies the Fisher–Dirichlet density block as the unique UIH realisation of the density sector.

Section 5 takes the hydrodynamic limit to overdamped Fokker–Planck equations, showing that the same Fisher–Dirichlet operator governs diffusion in the continuum.

Section 6 treats coherent GKLS models with genuine off-diagonal operators, showing that the density sector again closes to a classical Fisher generator while circulation and coherent effects live in the skew channel J.

Section 7 develops the UIH hypocoercivity theorem for finite dimensional generators, bounding the decay rate of K in terms of the Fisher gap and dimensionless UIH couplings built from J and $[G, J]$.

Section 8 introduces the UIH renormalisation group, preserves Fisher dissipation on chosen observables, and identifies a diffusive Fisher universality basin in the (λ_F, g_1, g_2) plane.

Section 9 presents numerical tests of decay clocks, Fisher floors and coupling-parameter behaviour across representative Markov, Fokker–Planck and GKLS examples.

Section 10 implements UIH channel tomography and universality spectroscopy on IBM Quantum hardware: process tomography of idle and driven channels is used to reconstruct K in the BKM geometry, split it into G and J, and estimate $(\lambda_F, \lambda_{\text{hyp}}, g_1, g_2)$ together with an emergent Kähler structure and holomorphicity defects. Section 11 summarises the UIH picture, discusses falsifiers, and outlines extensions to larger probe sectors and many-body systems.

2 Background from the reversible and dissipative Fisher geometry

This section gathers the parts of the companion work that will be used in the sequel. The aim is to make the present paper logically self contained for a reader who is willing to accept the main theorems of the previous studies, without repeating proofs. We therefore state the reversible Fisher Schrödinger structure on (ρ, S), the dissipative Fisher metriplectic structure on ρ, and the way in which both are organised by a single underlying Fisher information object.

Commentary. This section only fixes the geometric stage. There is one Fisher geometry on densities, and it supports two kinds of motion. The antisymmetric operator J generates the reversible Schrödinger flow at fixed entropy; the symmetric operator G generates the dissipative gradient flow that relaxes entropy. Later, when we look at Markov chains, Fokker-Planck equations and GKLS generators, we are simply realising these same two Fisher quadratures in concrete models.

2.1 Reversible Fisher Schrödinger sector

The reversible companion paper [1] works with hydrodynamic variables $\rho(x,t)$ and $S(x,t)$ on configuration space, treated as probability density and phase potential. Dynamics are generated by a Poisson bracket on functionals $F[\rho, S]$ of Dubrovin Novikov type, restricted by locality, Euclidean invariance, global phase symmetry, and reversibility. Within that class the classification reduces the bracket to the canonical form

$$\{F, G\} = \int \left(\frac{\delta F}{\delta \rho} \frac{\delta G}{\delta S} - \frac{\delta F}{\delta S} \frac{\delta G}{\delta \rho} \right) \mathrm{d}x,$$

up to equivalence inside the axioms. In particular, $\{\rho(x), S(y)\} = \delta(x - y)$.

The Hamiltonian functionals of interest are of the form

$$H[\rho, S] = \int \left(\frac{\rho |\nabla S|^2}{2m} + V(x)\rho + Q[\rho] \right) \mathrm{d}x,$$

where the first term is the usual kinetic energy in hydrodynamic variables, the second is a potential term, and $Q[\rho]$ is a local curvature functional built from ρ and its gradient.

The equations of motion are the continuity equation and a Hamilton Jacobi equation with an extra potential coming from Q. Reversibility and the projective linearity requirement at the complex level restrict Q to be the Fisher functional on ρ. Concretely, one finds that the only admissible local functional of the form

$$F[\rho] = \int f(\rho) |\nabla \rho|^2 \mathrm{d}x$$

whose Euler Lagrange contribution can be absorbed into a linear complex Schrödinger equation after a local complexifier $\psi = \sqrt{\rho} e^{\mathrm{i}S/\hbar}$ is the Fisher functional

$$F[\rho] = 4\alpha \int |\nabla \sqrt{\rho}|^2 \mathrm{d}x,$$

with $\alpha > 0$ playing the role of $\hbar^2/(2m)$. The associated potential

$$Q_\alpha(\rho) = -\alpha \frac{\Delta \sqrt{\rho}}{\sqrt{\rho}}$$

is then the usual quantum potential. With this choice, the equations of motion for (ρ, S) are equivalent to the linear Schrödinger equation for ψ, and the scale α is fixed by many body and symmetry arguments.

At this reversible level the Fisher object is therefore a curvature term in the Hamilton Jacobi equation. It is determined by axioms that know nothing about Lindblad operators or dissipation. It is precisely this object that will reappear in the dissipative and hydrodynamic sectors.

2.2 Dissipative Fisher metriplectic sector

The dissipative companion paper moves to a density only description. The state space is a set of strictly positive densities ρ on a domain Ω, with fixed total mass and admissible boundary classes (periodic or no flux).

A free energy functional $F[\rho]$ induces a chemical potential $\mu = \delta F/\delta \rho$ up to an additive constant. The irreversible part of the dynamics is assumed to be local and quadratic in $\nabla \mu$ at fixed ρ, with a symmetric positive definite mobility tensor $G(\rho, x)$ that is uniformly elliptic on the domain.

Under a short list of axioms, including a steepest descent principle at fixed state and locality of probes, one shows that the only possible irreversible drift is the continuity form

$$\partial_t \rho \big|_{\mathrm{irr}} = \nabla \cdot \big(\rho \, G(\rho, x) \, \nabla \mu(\rho) \big),$$

and that the instantaneous irreversible power

$$P_{\text{irr}}(\rho;\mu) = \frac{1}{2}\int \rho\, \nabla\mu \cdot G(\rho,x)\, \nabla\mu\, \mathrm{d}x$$

induces a weighted $H_\rho^{-1}(G)$ norm on conservative tangents v at fixed ρ. The associated entropy production is

$$\dot{\sigma}(\rho) = \int \rho\, \nabla\mu \cdot G(\rho,x)\, \nabla\mu\, \mathrm{d}x,$$

and a cost entropy inequality relates $\dot{\sigma}(\rho)$, the minimal quadratic cost $C_{\min}(\rho;v)$ to realise a given tangent v, and the instantaneous dissipation $\langle v,\mu\rangle$. A curvature coercivity bound further shows that the Hessian of F at ρ is bounded below by a curvature constant times the squared H_ρ^{-1} norm of v.

For later use it is convenient to record a mild interpretation of the cost–entropy relation as an instantaneous dissipative speed constraint. At a fixed state ρ the sharp inequality

$$\langle v,\mu\rangle^2 \;\leq\; 2\, C_{\min}(\rho;v)\, \dot{\sigma}(\rho)$$

shows that, for a prescribed entropy–production scalar $\dot{\sigma}(\rho)$, the free–energy decay rate $|\dot{F}| = |\langle v,\mu\rangle|$ cannot be increased independently of the minimal quadratic cost $C_{\min}$ of the flux realising v. The UIH irreversible drift is the unique direction that saturates this bound in the weighted $H_\rho^{-1}(G)$ geometry. We will only appeal to this as a local geometric way to read the inequality, rather than as a separate postulate about thermodynamic optimisation.

The reversible drift at the density level is represented by an antisymmetric operator $J(\rho,x)$ that generates fluxes of the form

$$j_{\text{rev}} = -\rho\, J(\rho,x)\, \nabla\mu,$$

subject to a weighted Liouville constraint $\nabla\cdot(\rho J) = 0$ and the no work condition $\int \mu\, \partial_t\rho|_{\text{rev}}\mathrm{d}x = 0$. Under these conditions the reversible sector is orthogonal to the irreversible cone in the H_ρ^{-1} inner product, and the instantaneous scalars $\dot{\sigma}$, $C_{\min}$ and the curvature constant depend only on G and F.

The Fisher information reappears in this dissipative sector as the curvature of F and as the metric underlying the H_ρ^{-1} geometry. In particular, when F is chosen to be a relative entropy functional with respect to a reference density and G is taken to be a scalar mobility, one recovers the standard Fokker Planck and Wasserstein gradient flow structures.

It is convenient to record at this point a simple entropy monotonicity identity that will be used implicitly later. When the physical evolution is chosen to be the G–gradient flow of the free energy F, so that

$$\partial_t\rho_t \;=\; \nabla\cdot\big(\rho_t\, G(\rho_t,x)\, \nabla\mu_t\big), \qquad \mu_t \;=\; \frac{\delta F}{\delta\rho}(\rho_t),$$

the free energy is automatically a Lyapunov functional. Differentiating $F[\rho_t]$ along the trajectory and using the continuity equation gives

$$\frac{d}{dt}F[\rho_t] \;=\; \int \mu_t\,\partial_t\rho_t\,dx \;=\; -\int \rho_t\,\nabla\mu_t\cdot G(\rho_t,x)\,\nabla\mu_t\,dx \;=\; -\dot{\sigma}(\rho_t) \;\le\; 0,$$

where $\dot{\sigma}(\rho_t)$ is the entropy production defined above. In particular, for the canonical choice of a relative entropy functional $F(\rho) = \int \rho\log(\rho/\rho_*)\,dx$ with respect to a fixed smooth reference density ρ_* and a scalar mobility $G = D\,\mathrm{Id}$, this recovers the familiar statement that relative Shannon entropy is nonincreasing along Fokker-Planck flows, with entropy production $\dot{\sigma}(\rho_t) = D\int |\nabla\rho_t|^2/\rho_t\,dx \ge 0$. Within the present axioms this "Second Law" is therefore nothing more than a geometric monotonicity property of the Fisher metric: irreversibility corresponds to a nondegenerate positive operator G on the tangent space, and the associated relative entropy functional decreases along its gradient flow.

The companion paper goes further and records detailed diagnostics and falsifiers, but for the present work it is enough to know that within the axioms the symmetric G and antisymmetric J are fixed up to simple gauges, and that the irreversible and reversible parts can be cleanly separated.

2.3 Common Fisher structure

Both companion papers [1, 2] are built around a single Fisher object. In the reversible Schrödinger sector Fisher curvature appears as a regulariser of the Hamilton Jacobi equation and is responsible for the quantum potential.

In the dissipative metriplectic sector Fisher information appears as the curvature of the free energy and as the metric that underlies the H_ρ^{-1} geometry and its cost entropy relations. The present paper does not introduce a new Fisher functional. It instead explores how these existing structures interact with Lindblad generators and hydrodynamic limits.

At a practical level this means that all constructions in the sequel will assume a fixed Fisher geometry on the density manifold. The symmetric mobility G is the same one that appears in the metriplectic study.

The antisymmetric operator J is the same one that appears in the reversible study, at least at the level of coarse grained densities. The new object $\mathcal{K} = G + \mathrm{i}J$ that will be introduced in the next section is therefore not an extra degree of freedom. It is a complex packing of existing operators that simplifies the bookkeeping when one wants to talk about reversible and irreversible quadratures in the same breath.

In addition, the same Fisher structure reduces to the usual parametric Fisher information for one dimensional translation families. Let ρ be a smooth, strictly positive density on $\mathbb{R}$ and consider the translations $\rho_\mu(x) = \rho(x-\mu)$. The associated configuration space Fisher functional in the density sector is

$$I_x[\rho] \;=\; \int_{\mathbb{R}} \rho(x)\,\big|\partial_x \log\rho(x)\big|^2\,dx,$$

which induces a one dimensional Fisher metric on the parameter μ. The companion

Fisher Kähler paper [3] shows that this metric coincides exactly with the parametric Fisher information

$$I_{\mathrm{param}}[\rho] \;=\; \int_{\mathbb{R}} \big(\partial_\mu \log \rho_\mu(x)\big)^2 \, \rho_\mu(x)\, dx,$$

so that $I_{\mathrm{param}}[\rho] = I_x[\rho]$ for translation channels. In the EPI language this identifies the UIH Fisher functional with the Fisher information used in Extreme Physical Information; in the present operator language it means that the single density sector Fisher geometry supporting the decomposition $K = G + iJ$ simultaneously controls configuration space curvature, entropy production, and parametric information for translation invariant channels.[1]

3 Information manifolds and UIH $\mathcal{K}$ flows

We now introduce the language in which the rest of the paper will be written.

We regard the set of states, classical or quantum, as a manifold equipped with an information metric and a complex mobility operator. The symmetric part of the mobility encodes irreversible gradient flow with respect to a free energy, the antisymmetric part encodes reversible Hamiltonian flow, and Lindblad generators appear as particular ways of coupling these parts when the states are density matrices rather than scalar densities.

3.1 Information manifolds

An information manifold is a smooth manifold $\mathcal{M}$ of states equipped with three pieces of structure. First, there is a free energy functional $F : \mathcal{M} \to \mathbb{R}$, usually a relative entropy or energy functional that plays the role of a Lyapunov function for the irreversible dynamics.

Second, there is an information metric g on the tangent bundle $T\mathcal{M}$, typically a Fisher type Riemannian metric that arises as the Hessian of F or of a related divergence. Third, there is a compatibility between F and g that allows one to define a gradient flow of F on $\mathcal{M}$ with respect to g.

In the classical density setting $\mathcal{M}$ is the space of strictly positive densities ρ with fixed mass on a domain Ω. The free energy $F[\rho]$ is often taken to be a relative entropy $\int \rho \log(\rho/\rho_*)\mathrm{d}x$ plus potential terms, and the Fisher metric can be realised through the weighted H_ρ^{-1} geometry defined by the elliptic operators L_ρ and $L_{\rho,G}$. In the quantum setting $\mathcal{M}$ can be taken as the convex set of density matrices on a Hilbert space. The role of F is played by the Umegaki relative entropy or a related quantity, and the Fisher metric becomes a quantum information metric on density matrices. For the explicit examples in this paper we will mostly work in the classical setting, but the quantum examples inherit the same logic.

[1] Throughout this paper, the complex form $K = G + iJ$ is used when acting on wavefunctions or density matrices, while the real metric-adjoint form $K = G + J$ (with J absorbing the factor of i) is used for perturbations in the metrised Hilbert space. The two conventions are reconciled explicitly in Section 9.

Commentary. Here we forget about PDEs and work in a finite-dimensional laboratory. We take a vector space with a metric M and split a linear operator K into a symmetric part G and a skew part J. The symmetric G is the "friction" or downhill part in the metric, the skew J is the "rotation" or sideways part, and $K = G + J$ just packages them together. All of the K-split statements in this section are linear algebra, but the same patterns will reappear for Markov chains, Fokker-Planck flows and GKLS semigroups.

On such a manifold the gradient $\nabla_g F$ is defined implicitly by the relation

$$g_\rho(\nabla_g F(\rho), v) = \mathrm{D}F(\rho)[v]$$

for all tangent vectors $v \in T_\rho \mathcal{M}$. In the density setting the metric g_ρ is represented in terms of the elliptic operators by

$$g_\rho(v, w) = \int \rho \, \nabla\phi_v \cdot G(\rho, x) \, \nabla\phi_w \, \mathrm{d}x,$$

where ϕ_v and ϕ_w are potentials solving Poisson type equations $L_{\rho,G}\phi_v = -v$, and similarly for w. The metriplectic paper shows that, under the stated axioms, the irreversible drift is exactly $-\nabla_g F$ and is represented as a continuity equation $\partial_t \rho = \nabla\cdot(\rho G \nabla\mu)$ with $\mu = \delta F/\delta\rho$.

3.2 Complex mobility operator

To prepare for the Lindblad discussion it is convenient to lift this gradient flow picture to the cotangent space. Instead of working directly with tangent vectors v, we consider covectors p that play the role of chemical potentials or Hamiltonian potentials. The metric g and its inverse identify tangent and cotangent spaces, and the irreversible drift can be written schematically as $v_{\mathrm{irr}} = Gp$ where p is the gradient of F and G is a positive selfadjoint operator with respect to g.

The reversible sector meanwhile is encoded by an antisymmetric operator J that maps covectors to tangent vectors in such a way that the power $\langle Jp, p\rangle$ vanishes for all p. In the density setting J is represented via the weighted Liouville identity and generates divergence free fluxes. In the quantum setting J corresponds to the commutator with a Hamiltonian and generates unitary evolution that preserves the von Neumann entropy.

The complex mobility operator $\mathcal{K}$ is then defined by

$$\mathcal{K} = G + \mathrm{i}J.$$

Acting on a complex potential

$$U = \mu + \mathrm{i}\psi$$

with μ the chemical potential and ψ a Hamiltonian potential, $\mathcal{K}$ produces a complex current of the form

$$v = \Re(\mathcal{K}U)$$

which can then be inserted into a continuity equation. In the density case this gives

$$\partial_t \rho = -\nabla \cdot j, \qquad j = \rho \, \Re(\mathcal{K} \nabla U),$$

with j decomposed into reversible and irreversible parts. In the Schrödinger case, after the complexifier, the same structure can be read at the level of the wave function. In the Lindblad case, $\mathcal{K}$ acts on matrix valued potentials and the continuity equation is replaced by the GKLS master equation.

The usefulness of $\mathcal{K}$ is twofold. Conceptually, it packages the symmetric and antisymmetric mobility into a single object that carries both quadratures. Practically, it allows one to write diagnostics and inequalities in a compact way. For example, the cost entropy inequality and curvature coercivity bounds of the metriplectic paper can be viewed as statements about the real part of $\mathcal{K}$ on the real axis of potentials, while reversible invariance corresponds to the imaginary part dropping out of certain scalar functionals.

In linear response one can form, for suitable observables, a susceptibility

$$\chi(\omega) \;=\; \big\langle A,\, (i\omega I - K)^{-1} B \big\rangle_g,$$

where the inner product is taken with respect to the metric g and A, B encode the chosen probe and readout. Whenever the associated semigroup is causal and $\chi(\omega)$ extends analytically to the upper half plane, standard dispersion theory implies a Kramers-Kronig relation between the real and imaginary parts of χ. In the present split this means that the dissipative and reversible quadratures encoded by G and J appear as the two Hilbert transform linked components of a single causal response function rather than as freely tunable additions. The concrete density sector tests in the metriplectic companion paper provide one instance of this general picture.

3.3 Schrödinger, Fokker Planck, and Lindblad as $\mathcal{K}$ flows

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Within this abstract language, the three families of dynamics that concern us can be described informally as follows.

First, Schrödinger evolution on pure states corresponds to a purely imaginary $\mathcal{K}$ with $G = 0$ and J given by the Hamiltonian commutator in the appropriate representation. The free energy is constant, and the state moves along level sets of F. At the hydrodynamic level this structure is encoded by the reversible companion paper in the canonical bracket on (ρ, S) and the Fisher curvature.

Second, Fokker Planck evolution in the overdamped regime corresponds to a purely real $\mathcal{K}$ with $J = 0$ and G given by the Fisher metriplectic mobility. The state moves down the gradient of F with respect to the $H^{-1}_{\rho}(G)$ metric. The dissipative companion paper characterises this class in detail and shows that, under the stated axioms, the irreversible flow is uniquely determined by G and F.

Third, Lindblad evolution on density matrices mixes both parts. In the GKLS representation the generator of a contractive semigroup is a sum of a Hamiltonian commutator and a dissipator built from jump operators. When the jump operators and stationary state obey detailed balance, the dissipator can be interpreted as a gradient flow of the quantum relative entropy with respect to a quantum Fisher metric, while the Hamiltonian part is reversible.

In the special class of models studied in this paper the dissipator reduces on the diagonal to a classical Fisher gradient flow of the Kullback Leibler divergence, and the Hamiltonian part becomes a familiar reversible operator. The resulting dynamics on the diagonal sector can then be seen as a projection of a $\mathcal{K}$ flow compatible with the classical Fisher geometry.

The rest of the paper will make these schematic statements precise for a concrete family of GKLS generators with diagonal jumps and for their hydrodynamic limits.

Section 4 will introduce the finite dimensional GKLS model, show explicitly how its diagonal sector reduces to a reversible Markov chain, and identify the equality of quantum and classical entropy decay curves.

Section 5 will construct the hydrodynamic limit and show that the limiting Fokker Planck equation is exactly a Fisher metriplectic gradient flow. Section 6 will use a simple coherent two level GKLS model to illustrate how J feeds into an effective G on coarse grained populations in a strong dephasing regime. The present section is therefore the conceptual backbone that allows these examples to be read as instances of a single universal information hydrodynamics. This finite dimensional picture serves as a laboratory for the UIH K split.

In the next section, we embed it into diagonal GKLS semigroups whose density sector is exactly Fisher-Dirichlet, while in Section 6 we show that the same split arises in coherent GKLS models once the BKM metric at the stationary state is used.

The continuum Fokker-Planck flows of Section 5 and the numerical and hardware tests in the appendices then demonstrate that the same structures control irreversible response from classical overdamped Langevin dynamics all the way to real, physical noisy, superconducting qubits.

4 Finite GKLS with diagonal jumps and classical reversible chains

In this section we give a fully explicit family of finite dimensional GKLS generators that act as Lindblad semigroups on density matrices and reduce exactly to classical reversible Markov chains on their diagonal sector.

For diagonal states, the quantum relative entropy with respect to the stationary state coincides identically with the classical Kullback Leibler divergence, so the entropy decay curve of the GKLS semigroup on that sector is exactly the same as that of the corresponding Markov chain. This realises the discrete Fisher gradient flow on the probability simplex as a special case of a quantum information hydrodynamics, in a way that is completely transparent at the level of matrix elements and supported numerically by the script `01_gkls_diagonal_to_markov_checks.py`.

Commentary. This section answers a concrete question: what does the UIH picture look like for an honest Lindblad jump model? We start from a thermal GKLS generator, restrict the dissipator to the energy eigenbasis to obtain a classical Markov generator Q_{markov}, and then symmetrise with respect to the stationary law to obtain the Dirichlet operator G. The Fisher metric and entropy decay in the diagonal sector are not fitted by hand; they are determined entirely by the GKLS data.

4.1 Set up and notation

Let $\mathcal{H} = \mathbb{C}^N$ be a finite dimensional Hilbert space with orthonormal basis $\{|i\rangle\}_{i=1}^N$. A density matrix $\hat\rho$ is a positive semidefinite operator on $\mathcal{H}$ with unit trace. We will work in the matrix representation of $\hat\rho$ in the basis $|i\rangle$:

$$\hat\rho = \sum_{m,n=1}^{N} \rho_{mn}|m\rangle\langle n|, \qquad \rho_{mn} = \langle m|\hat\rho|n\rangle.$$

Fix a strictly positive probability vector $\pi = (\pi_1, \dots, \pi_N)$ with $\sum_i \pi_i = 1$. Think of π as a discrete Gibbs distribution or stationary measure.

We introduce non negative rates $k_{ij} \geq 0$ for all ordered pairs $i \neq j$. These rates will act as the jump intensities of a continuous time Markov chain in the classical picture, and as the amplitudes of Lindblad jump operators in the quantum picture. A convenient way to enforce detailed balance with respect to π is to start from a symmetric matrix $a_{ij} = a_{ji} \geq 0$ with zero diagonal and set

$$k_{ij} = a_{ij}\sqrt{\frac{\pi_j}{\pi_i}}, \qquad i \neq j.$$

Then

$$\pi_i k_{ji} = \pi_i a_{ji}\sqrt{\frac{\pi_i}{\pi_j}} = a_{ij}\sqrt{\pi_i\pi_j} = \pi_j a_{ij}\sqrt{\frac{\pi_j}{\pi_i}} = \pi_j k_{ij},$$

so detailed balance $\pi_i k_{ji} = \pi_j k_{ij}$ holds automatically.

On the quantum side we define Lindblad jump operators

$$L_{ij} = \sqrt{k_{ij}}\ |i\rangle\langle j|, \qquad i \neq j,$$

and we consider a GKLS generator with these jumps and no Hamiltonian term. That is, the evolution of a density matrix $\hat\rho_t$ is governed by

$$\frac{\mathrm{d}}{\mathrm{d}t}\hat\rho_t = \mathcal{L}(\hat\rho_t) := \sum_{i\neq j}\Big(L_{ij}\hat\rho_t L_{ij}^\dagger - \tfrac{1}{2}\{L_{ij}^\dagger L_{ij}, \hat\rho_t\}\Big).$$

We will show that this semigroup has a unique stationary state

$$\hat{\pi} = \sum_{i=1}^{N} \pi_i |i\rangle\langle i|,$$

and that its action on diagonal density matrices coincides exactly with that of a reversible Markov chain with generator built from $\{k_{ij}\}$.

4.2 Diagonal sector and reversible Markov chain

We first show that the diagonal and off diagonal matrix elements of $\hat{\rho}_t$ evolve in a simple and decoupled way under (4.1). Using the definitions (4.1) and (4.1), one checks

$$L_{ij}^{\dagger} L_{ij} = k_{ij}\, |j\rangle\langle j|.$$

Writing $\hat{\rho} = \sum_{m,n} \rho_{mn} |m\rangle\langle n|$, a short computation shows that the diagonal entries satisfy

$$\dot{\rho}_{ii} = \sum_{j \neq i} \big(k_{ij}\rho_{jj} - k_{ji}\rho_{ii} \big),$$

while the off diagonal entries satisfy

$$\dot{\rho}_{mn} = -\frac{1}{2}\rho_{mn} \sum_{l \neq m} k_{lm} - \frac{1}{2}\rho_{mn} \sum_{l \neq n} k_{ln}, \qquad m \neq n.$$

The derivation is standard: the term $L_{ij}\hat{\rho}L_{ij}^{\dagger}$ only contributes to diagonal entries, and the anticommutator term involves projectors $|j\rangle\langle j|$ that act diagonally in the chosen basis.

Equation (4.2) shows that the off diagonal entries decay exponentially to zero with a rate bounded below by half the total outgoing rate from the corresponding indices. They do not feed back into the diagonal entries. Equation (4.2) shows that the diagonal entries are closed under the evolution.

If we define the population vector $p(t)$ by

$$p_i(t) := \rho_{ii}(t) = \langle i|\hat{\rho}_t|i\rangle,$$

then (4.2) can be written as

$$\dot{p}_i(t) = \sum_{j \neq i} \big(k_{ij} p_j(t) - k_{ji} p_i(t) \big).$$

This is exactly the master equation of a continuous time Markov chain on $\{1, \ldots, N\}$, with transition rates k_{ij} from j to i. If we define a generator Q by

$$Q_{ij} = k_{ij}, \quad i \neq j, \qquad Q_{ii} = -\sum_{j \neq i} k_{ji},$$

then (4.2) can be written as

$$\dot{p}(t) = Q^T p(t),$$

where the column vector p has entries p_i. The detailed balance identity (4.1) is equivalent to

$$\pi_i Q_{ij} = \pi_j Q_{ji},$$

so Q is a reversible generator with invariant distribution π.

We also note that the stationary quantum state $\hat{\pi}$ satisfies $\mathcal{L}(\hat{\pi}) = 0$. Indeed, in the basis $|i\rangle$ we have

$$\hat{\pi} = \sum_i \pi_i |i\rangle\langle i|,$$

and using detailed balance each term in (4.1) cancels when applied to $\hat{\pi}$, in a manner analogous to the usual classical detailed balance identity. Thus $\hat{\pi}$ is the unique stationary state of $\mathcal{L}$ in the full quantum system, and its diagonal entries are the unique stationary distribution π of the classical chain.

4.3 Quantum relative entropy and classical Kullback Leibler divergence

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For a general density matrix $\hat{\rho}$ the Umegaki quantum relative entropy with respect to $\hat{\pi}$ is defined by

$$S(\hat{\rho}\|\hat{\pi}) = \mathrm{Tr}(\hat{\rho}(\log\hat{\rho} - \log\hat{\pi})).$$

When both $\hat{\rho}$ and $\hat{\pi}$ are diagonal in the same basis, the logarithms act elementwise, so if $\hat{\rho} = \sum_i p_i|i\rangle\langle i|$ and $\hat{\pi} = \sum_i \pi_i|i\rangle\langle i|$, then

$$S(\hat{\rho}\|\hat{\pi}) = \sum_{i=1}^{N} p_i(\log p_i - \log\pi_i) = \sum_{i=1}^{N} p_i \log\frac{p_i}{\pi_i} =: D_{\mathrm{KL}}(p\|\pi),$$

the classical Kullback Leibler divergence from p to π.

Suppose now that the initial state $\hat{\rho}_0$ is diagonal, so that $\hat{\rho}_t$ remains diagonal under the GKLS evolution (4.1) and the corresponding population vector $p(t)$ evolves by (4.2). Then for all times we have

$$S(\hat{\rho}_t\|\hat{\pi}) = D_{\mathrm{KL}}(p(t)\|\pi).$$

The decay of quantum relative entropy along the diagonal sector of the Lindblad semigroup is therefore identical, pointwise in time, to the decay of classical relative entropy along the corresponding Markov chain.

The evolution (4.2) is generated by a reversible Q, so it admits a well known Dirichlet form representation. One can show that

$$\frac{\mathrm{d}}{\mathrm{d}t} D_{\mathrm{KL}}(p(t)\|\pi) = -\frac{1}{2}\sum_{i,j=1}^{N} \pi_i k_{ji}\left(\frac{p_i}{\pi_i} - \frac{p_j}{\pi_j}\right)\left(\log\frac{p_i}{\pi_i} - \log\frac{p_j}{\pi_j}\right).$$

Every term in the sum is non negative by the usual monotonicity of the logarithm, so the derivative is non positive, and is zero if and only if p_i/π_i is constant in i. Thus

D_{KL} decays strictly along the Markov chain unless $p = \pi$, and the same holds for the quantum relative entropy of diagonal states under the GKLS evolution.

In the language of the metriplectic companion paper, (4.3) expresses the entropy production as a discrete Fisher quadratic form in the logarithmic gradient of p/π. Indeed, the difference $p_i/\pi_i - p_j/\pi_j$ is a discrete gradient in the tilted variable, and the factor $\pi_i k_{ji}$ is a discrete mobility. The entire finite dimensional picture is thus a discrete counterpart of the Fisher metric and H_ρ^{-1} geometry that appear in the continuum.

4.4 Numerical verification

The script `01_gkls_diagonal_to_markov_checks.py` implements this construction for concrete choices of N, π and k_{ij}. It constructs the Lindblad superoperator $\mathcal{L}$ in vectorised form, builds the Markov generator Q, and evolves both the GKLS equation and the Markov master equation from the same diagonal initial state. At a fixed set of times it compares:

- the population vectors extracted from the diagonal of $\hat{\rho}_t$ and from the Markov chain, and
- the relative entropies $S(\hat{\rho}_t \| \hat{\pi})$ and $D_{\mathrm{KL}}(p(t) \| \pi)$.

The script reports the maximum difference in populations and in relative entropy over the time grid. In all tested cases these differences are at the level of numerical integration error, confirming (4.2) and (4.3) at the level of floating point computations. For completeness, the script also checks positivity and normalisation of the density matrix along the evolution.

5 Hydrodynamic limit and Fisher metriplectic Fokker Planck flows

We now move from finite dimension to continuum. We keep the classical reversible chain constructed above but embed it in a family indexed by a lattice spacing a and endowed with a nearest neighbour structure. In the limit $a \to 0$ the master equation of the chain converges to an overdamped Fokker Planck equation with drift determined by a potential V and diffusion coefficient D. We then show that this Fokker Planck equation is exactly a Fisher metriplectic gradient flow with mobility tensor $G = DI$ and free energy $F[\rho] = \int \rho \log(\rho/\pi) \mathrm{d}x$, as described in the dissipative companion paper. Scripts `02_markov_to_fp_limit_checks.py` and `03_fp_fisher_metriplectic_checks.py` provide numerical support.

> **Commentary.** The finite reversible chain from Section 4 is now sent to a continuum limit. Jumps turn into drift and diffusion, and the discrete Fisher-Dirichlet form becomes the quadratic form of an overdamped Langevin / Fokker-Planck operator. The same Fisher geometry and K-split survive the limit. In the weighted H_ρ^{-1} Fisher geometry the continuum Fokker-Planck operator again takes the form $K = G + J$, with G giving entropy production and J giving incompressible transport.

5.1 Reversible chain on a lattice

For simplicity we work in one spatial dimension and with a periodic or reflecting domain. Fix a macroscopic interval $[-L, L]$ and a lattice spacing $a > 0$, and let

$$x_i = -L + ia, \qquad i = 0, 1, \dots, N-1,$$

where $Na = 2L$. We consider indices modulo N in the periodic case.

Let $V \colon [-L, L] \to \mathbb{R}$ be a smooth potential, and fix a diffusion coefficient $D > 0$. Define the discrete Gibbs weights

$$\pi_i^{(a)} = \frac{1}{Z_a} \exp\big(-V(x_i)/D\big), \qquad Z_a = \sum_{i=0}^{N-1} \exp\big(-V(x_i)/D\big).$$

We will write π_i when the dependence on a is clear from context.

We now define nearest neighbour rates that satisfy detailed balance with respect to π and have the natural diffusive scaling in a. For each i set

$$k_{i,i+1} = \frac{D}{a^2}\sqrt{\frac{\pi_{i+1}}{\pi_i}}, \qquad k_{i+1,i} = \frac{D}{a^2}\sqrt{\frac{\pi_i}{\pi_{i+1}}}.$$

This is the one dimensional specialisation of (4.1). It satisfies detailed balance in the sense that

$$\pi_i k_{i+1,i} = \pi_{i+1} k_{i,i+1} = \frac{D}{a^2}\sqrt{\pi_i \pi_{i+1}}.$$

The generator $Q^{(a)}$ of the chain is then

$$Q^{(a)}_{i,i+1} = k_{i,i+1}, \quad Q^{(a)}_{i,i-1} = k_{i,i-1}, \quad Q^{(a)}_{ii} = -k_{i+1,i} - k_{i-1,i},$$

with all other entries zero. The master equation for the population vector $p^{(a)}(t) = (p_i^{(a)}(t))_{i=0}^{N-1}$ is

$$\dot{p}_i^{(a)}(t) = k_{i,i+1} p_{i+1}^{(a)}(t) + k_{i,i-1} p_{i-1}^{(a)}(t) - (k_{i+1,i} + k_{i-1,i}) p_i^{(a)}(t).$$

The stationary distribution of this chain is $\pi^{(a)}$, and detailed balance ensures reversibility.

5.2 Flux form and continuum limit

To extract a continuum equation, it is convenient to rewrite (5.1) in flux form. Define the flux across the bond between sites i and $i+1$ by

$$J_{i+1/2} = k_{i+1,i} p_i^{(a)} - k_{i,i+1} p_{i+1}^{(a)}.$$

Then (5.1) can be written as

$$\dot{p}_i^{(a)} = J_{i-1/2} - J_{i+1/2}.$$

Introduce the tilted variables

$$f_i^{(a)}(t) = \frac{p_i^{(a)}(t)}{\pi_i},$$

so that $p_i^{(a)} = f_i^{(a)}\pi_i$. Using (5.1) and detailed balance, we can write the flux as

$$\begin{aligned} J_{i+1/2} &= \frac{D}{a^2}\sqrt{\frac{\pi_i}{\pi_{i+1}}}\, p_i^{(a)} - \frac{D}{a^2}\sqrt{\frac{\pi_{i+1}}{\pi_i}}\, p_{i+1}^{(a)} \\ &= \frac{D}{a^2}\Big(\sqrt{\frac{\pi_i}{\pi_{i+1}}}\,\pi_i f_i^{(a)} - \sqrt{\frac{\pi_{i+1}}{\pi_i}}\,\pi_{i+1} f_{i+1}^{(a)}\Big) \\ &= \frac{D}{a^2}\sqrt{\pi_i\pi_{i+1}}\,(f_i^{(a)} - f_{i+1}^{(a)}). \end{aligned}$$

This is a discrete gradient in $f^{(a)}$ with a mobility prefactor $\sqrt{\pi_i\pi_{i+1}}$.

We now define a piecewise constant density $\rho^{(a)}(x,t)$ by

$$\rho^{(a)}(x_i,t) = \frac{p_i^{(a)}(t)}{a},$$

so that $\sum_i p_i^{(a)}(t) = 1$ approximates $\int \rho^{(a)}(x,t)\mathrm{d}x = 1$. Similarly define $\pi^{(a)}(x_i) = \pi_i/a$, so that $\pi^{(a)}$ approaches the continuum Gibbs density $\pi(x) \propto \exp(-V(x)/D)$ as $a \to 0$.

In the formal limit $a \to 0$, one shows that

$$\sqrt{\pi_i\pi_{i+1}} \approx a\,\pi(x_{i+1/2}),$$

where $x_{i+1/2} = x_i + a/2$, and

$$f_i^{(a)} - f_{i+1}^{(a)} \approx -a\,\partial_x f(x_{i+1/2}),$$

where $f(x,t) = \rho(x,t)/\pi(x)$. Thus

$$J_{i+1/2} \approx -\frac{D}{a}\,\pi(x_{i+1/2})\,\partial_x f(x_{i+1/2}).$$

Using (5.2) and dividing by a we have

$$\partial_t\rho^{(a)}(x_i,t) = \frac{1}{a}\dot{p}_i^{(a)} = \frac{1}{a}(J_{i-1/2} - J_{i+1/2}) \approx -\partial_x J(x_i,t),$$

where in the limit we can identify

$$J(x,t) = -D\,\pi(x)\,\partial_x f(x,t) = -D\,\big(\partial_x\rho(x,t) - \rho(x,t)\,\partial_x \log\pi(x)\big).$$

The continuum equation is therefore

$$\partial_t \rho(x,t) = -\partial_x J(x,t) = \partial_x\big(D\,\partial_x \rho + D\,\rho\,\partial_x \log \pi\big).$$

Since $\log \pi(x) = -V(x)/D + \text{const}$, we have $\partial_x \log \pi = -V'(x)/D$, so

$$\partial_t \rho = \partial_x\big(D\,\partial_x \rho - \rho V'\big) = \partial_x\big(\rho V'\big) + D\,\partial_{xx}\rho.$$

This is the standard overdamped Fokker Planck equation with drift potential V and diffusion coefficient D.

The same argument extends to higher dimensions and more general neighbour graphs. The key point is that the diffusive scaling $k_{ij} \sim Da^{-2}$ and the detailed balance structure ensure that the macroscopic limit is a second order operator of the form $\nabla \cdot (\rho \nabla V + D\nabla\rho)$, which is the continuum Fisher metriplectic drift with $G = DI$ and free energy built from the Gibbs density.

5.3 Fokker Planck as a Fisher metriplectic K flow

We now identify the Fokker Planck equation (5.2) with a Fisher metriplectic K flow in the sense of the dissipative companion paper. The generalisation to higher dimensions is straightforward.

Let $\rho(x,t)$ be a smooth strictly positive density on $[-L, L]$ and define the continuum Gibbs density

$$\pi(x) = \frac{1}{Z} e^{-V(x)/D}, \qquad Z = \int_{-L}^{L} e^{-V(x)/D} \mathrm{d}x.$$

Define the free energy functional

$$F[\rho] = \int_{-L}^{L} \rho(x)\ \log \frac{\rho(x)}{\pi(x)}\,\mathrm{d}x.$$

A variation $\rho \mapsto \rho + \varepsilon\eta$ with $\int \eta = 0$ yields

$$\delta F = \int \eta(x)\left(\log\rho(x) + 1 + \frac{V(x)}{D}\right)\mathrm{d}x,$$

so the chemical potential is

$$\mu(x) = \frac{\delta F}{\delta\rho(x)} = \log\rho(x) + \frac{V(x)}{D} + \text{const}.$$

The constant does not affect gradients and can be ignored.

We choose the mobility tensor $G = D$ acting as a scalar multiple of the identity. In one dimension this gives

$$L_{\rho,G}\phi = -\partial_x(\rho D \partial_x \phi).$$

The irreversible Fisher metriplectic drift is then

$$\partial_t \rho = \partial_x \big(\rho D \partial_x \mu\big).$$

Using (5.3), we have

$$\partial_x \mu = \frac{\partial_x \rho}{\rho} + \frac{1}{D} V',$$

so

$$\rho D \partial_x \mu = D\, \partial_x \rho + \rho V'.$$

Inserting this into (5.3) gives exactly (5.2). Thus the Fokker Planck equation obtained from the hydrodynamic limit of the reversible chains is exactly the Fisher metriplectic gradient flow of F with mobility $G = D$, in the class singled out by the dissipative companion paper.

The entropy production along the flow is

$$\frac{\mathrm{d}}{\mathrm{d}t} F[\rho_t] = \int \mu \partial_t \rho \,\mathrm{d}x = \int \mu\, \partial_x (\rho D \partial_x \mu)\,\mathrm{d}x = -\int \rho D (\partial_x \mu)^2 \mathrm{d}x,$$

assuming suitable boundary conditions. This is minus the square of the $H^{-1}_{\rho}(G)$ norm of μ, and coincides with the continuum limit of the discrete Dirichlet form (4.3). In particular, the decay of F along the Fokker Planck flow is the continuum counterpart of the decay of Kullback Leibler divergence along the reversible chain, and the discrete Fisher form in (4.3) converges to the continuum Fisher form $\int \rho D |\partial_x \mu|^2 dx$.

5.4 Numerical verification

The script `02_markov_to_fp_limit_checks.py` implements the reversible chains described above for a sequence of lattice spacings and compares them to a numerical solution of the Fokker Planck equation (5.2). For each spacing a it:

- constructs $Q^{(a)}$ and evolves the master equation (5.1) for a fixed initial density profile,
- interpolates the resulting discrete density $\rho^{(a)}(x,t)$ to a common grid, and
- compares it to the Fokker Planck solution $\rho(x,t)$ at the same times.

The code reports error norms $E(a)$ between $\rho^{(a)}$ and ρ and estimates a convergence rate in a using a log log fit. In all tested regimes the errors decay at the expected order dictated by the spatial discretisation, and the discrete relative entropy $F[p^{(a)}]$ decays along curves that converge to the continuum decay of $F[\rho]$.

The script `03_fp_fisher_metriplectic_checks.py` works directly at the continuum level. It discretises the Fokker Planck equation (5.2) on a fine grid and, at each time step, computes:

- a finite difference approximation to $\partial_t \rho$,
- the Fisher metric right hand side $\partial_x(\rho D \partial_x \mu)$, and
- the free energy $F[\rho]$ and the quadratic form $\int \rho D (\partial_x \mu)^2 dx$.

It then checks numerically that $\partial_t \rho$ matches $\partial_x(\rho D \partial_x \mu)$ to within discretisation error, and that $\mathrm{d}F/\mathrm{d}t \approx -\int \rho D (\partial_x \mu)^2 dx$ along the evolution. These diagnostics provide a concrete numerical confirmation of the Fisher metriplectic structure at the PDE level.

Together, the reversible GKLS to Markov reduction of Section 4 and the hydrodynamic Fisher Fokker Planck limit of this section show that the entropy decay and Fisher geometry of a simple class of Lindblad semigroups can be understood entirely in terms of classical Fisher gradient flows on densities, once one restricts to the diagonal sector and passes to the macroscopic scale.

6 Coherent GKLS models, coherences, and effective Fisher chains

The examples so far have been deliberately classical in flavour. Lindblad generators with diagonal jump operators and no Hamiltonian term act as quantum lifts of reversible Markov chains, and their entropy decay on the diagonal sector is exactly classical. In this section we consider a genuinely coherent GKLS model in which the Hamiltonian and the dissipator are not simultaneously diagonal, so that populations and coherences interact. At the level of the full density matrix the evolution is first order in time and of GKLS type, but at the level of populations alone the dynamics becomes second order and non Markovian. In a strong dephasing regime one can nevertheless eliminate the coherences and obtain an effective classical Markov chain for the populations. This effective chain falls inside the Fisher metriplectic class, and its rates depend on the Hamiltonian part in a way that makes the interplay between J and G explicit.

Commentary. Up to now the dissipative sector came from GKLS generators that were already diagonal in the energy basis. Here we let a genuinely coherent GKLS model run and ask what the diagonal density alone "sees". The off-diagonal coherences feed into an effective Markov generator on the populations, and its symmetric block is still a Fisher-Dirichlet operator. In UIH language, the coherent J sector renormalises the effective G seen by the coarse-grained density, but it does not destroy the underlying Fisher-metriplectic structure.

6.1 A qubit GKLS model with coherences

We work with a single qubit and choose a basis in which the Lindblad operator is diagonal but the Hamiltonian is not. Let $\{|0\rangle, |1\rangle\}$ be the eigenbasis of σ_z, and set

$$H = \frac{\omega}{2}\,\sigma_x, \qquad L = \sqrt{\gamma}\,\sigma_z,$$

with real parameters $\omega > 0$ and $\gamma > 0$. The GKLS equation reads

$$\frac{\mathrm{d}}{\mathrm{d}t}\hat{\rho}_t = -\mathrm{i}[H, \hat{\rho}_t] + L\hat{\rho}_t L^\dagger - \frac{1}{2}\{L^\dagger L, \hat{\rho}_t\}.$$

This generator has a unique stationary state $\hat{\rho}_* = \frac{1}{2}I$ and describes coherent rotations about the x axis combined with dephasing in the σ_z basis.

It is convenient to write the state in Bloch form,

$$\hat{\rho}_t = \frac{1}{2}\big(I + x(t)\sigma_x + y(t)\sigma_y + z(t)\sigma_z\big),$$

where the real vector (x, y, z) lies in the unit ball. Inserting this parameterisation into (6.1) and using the usual Pauli algebra, one finds that the Bloch components satisfy the linear system

$$\dot{x} = -2\gamma\, x,$$
$$\dot{y} = -2\gamma\, y - \omega z,$$
$$\dot{z} = \omega y.$$

The dissipator damps the x and y components at rate 2γ and leaves z unchanged, while the Hamiltonian couples y and z through precession at frequency ω. The stationary point is at $x = y = z = 0$, corresponding to the maximally mixed state $\hat{\rho}_*$.

The population of the excited state $|1\rangle$ is

$$p_1(t) = \langle 1|\hat{\rho}_t|1\rangle = \frac{1 + z(t)}{2},$$

and the ground state population is $p_0(t) = 1 - p_1(t) = \frac{1-z(t)}{2}$. The diagonal sector is therefore determined by the single function $z(t)$.

Combining (6.1) and (6.1) we can eliminate y. Differentiating (6.1) gives

$$\ddot{z} = \omega\dot{y} = \omega(-2\gamma y - \omega z) = -2\gamma\omega y - \omega^2 z.$$

Using $\omega y = \dot{z}$ from (6.1), we obtain

$$\ddot{z} + 2\gamma\dot{z} + \omega^2 z = 0.$$

This is the equation of a damped harmonic oscillator with natural frequency ω and damping coefficient 2γ. In terms of the population p_1 we have

$$\ddot{p}_1 + 2\gamma\dot{p}_1 + \omega^2\Big(p_1 - \frac{1}{2}\Big) = 0.$$

Thus, although the full GKLS equation (6.1) is first order in time, the induced dynamics on the coarse grained population variable p_1 is second order and involves an inertial term $\ddot{p}_1$. There is no exact closed first order Markovian equation for p_1 alone at finite γ; any attempt to write $\dot{p}_1$ as a function of p_1 only would require keeping track of hidden variables that encode the coherence y.

6.2 Overdamped limit and effective two state chain

In the strong dephasing regime $\gamma \gg \omega$, the variables x and y are fast and strongly damped, while z evolves slowly. On timescales large compared to γ^{-1} but not so large that z has fully relaxed, the system is effectively overdamped and one can eliminate y adiabatically from the dynamics.

Formally, on the slow manifold one can set $\dot{y} \approx 0$ in (6.1) and solve for y in terms of z:

$$0 \approx -2\gamma y - \omega z \quad \Longrightarrow \quad y \approx -\frac{\omega}{2\gamma} z.$$

Substituting this into (6.1) gives an effective first order equation for z:

$$\dot{z} \approx \omega y \approx -\frac{\omega^2}{2\gamma} z.$$

Equivalently,

$$\dot{z} = -\kappa z + r(t), \qquad \kappa = \frac{\omega^2}{2\gamma},$$

where $r(t)$ is a small residual term that vanishes in the asymptotic regime $\gamma \to \infty$ at fixed ω. In terms of the population p_1 this reads

$$\dot{p}_1 = \frac{1}{2}\dot{z} \approx -\frac{\omega^2}{4\gamma}(2p_1 - 1) = -\frac{\omega^2}{2\gamma}\Big(p_1 - \frac{1}{2}\Big).$$

This is exactly the master equation of a symmetric two state Markov chain with states $|0\rangle$ and $|1\rangle$, stationary distribution $\pi_0 = \pi_1 = 1/2$, and transition rates

$$k_{01} = k_{10} = \frac{\omega^2}{4\gamma}.$$

Indeed, the master equation for such a chain is

$$\dot{p}_1 = -k_{10}p_1 + k_{01}p_0 = -k_{10}p_1 + k_{01}(1 - p_1) = -2k_{10}\Big(p_1 - \frac{1}{2}\Big),$$

so identifying coefficients gives $2k_{10} = \omega^2/(2\gamma)$ and hence $k_{10} = \omega^2/(4\gamma)$.

The effective generator

$$Q_{\text{eff}} = \begin{pmatrix} -k & k \\ k & -k \end{pmatrix}, \qquad k = \frac{\omega^2}{4\gamma},$$

is reversible with respect to $\pi = (1/2, 1/2)$. The free energy functional on the population simplex is the relative entropy

$$F[p] = \sum_{i=0}^{1} p_i \log \frac{p_i}{\pi_i} = (1 - p_1)\log \frac{1 - p_1}{1/2} + p_1 \log \frac{p_1}{1/2}.$$

Writing $p = p_1$ and suppressing the time argument, we can calculate

$$\frac{\mathrm{d}F}{\mathrm{d}p} = -\log \frac{1-p}{1/2} - 1 + \log \frac{p}{1/2} + 1 = \log \frac{p}{1-p}.$$

Thus the discrete chemical potential is

$$\mu(p) = \frac{\mathrm{d}F}{\mathrm{d}p} = \log \frac{p}{1-p}.$$

The effective Markov dynamics for p can then be written as a one dimensional gradient flow

$$\dot{p} = -M(p)\,\mu(p),$$

with mobility

$$M(p) = \frac{\omega^2}{2\gamma} \frac{p - 1/2}{\log(p/(1-p))}.$$

For $p \in (0, 1)$ and $p \neq 1/2$ the numerator and denominator have the same sign, so $M(p) > 0$. At the symmetric point $p = 1/2$ one can use l'Hospital's rule to define the limit

$$\lim_{p \to 1/2} \frac{p - 1/2}{\log(p/(1-p))} = \frac{1}{4},$$

so M extends smoothly to a strictly positive function on the open interval. The dynamics (6.2) is therefore a Fisher type one dimensional gradient flow of F with a state dependent scalar mobility $M(p)$, entirely analogous to the continuum Fisher metriplectic flows considered in the dissipative companion paper.

This example shows that even a tiny quantum system with genuine coherences produces, in a suitable regime, an effective classical chain whose entropy decay and metric structure sit inside the Fisher metriplectic class. The reversible Hamiltonian part J does not appear directly in the irreversible entropy production, but it does feed into the effective mobility through the factor ω^2/γ in (6.2).

6.3 Numerical illustration

The script `04_gkls_coherence_elimination_checks.py` implements the qubit GKLS model (6.1) in the Bloch representation and compares the exact GKLS evolution to the effective Markov chain dynamics in the overdamped regime.

Concretely, the script:

- integrates the linear system (6.1) to (6.1) for chosen values of ω and γ with $\gamma \gg \omega$, from an initial state with non equilibrium populations and nonzero coherences;
- extracts the population $p_1(t)$ and fits its long time decay to a single exponential, obtaining an effective rate $\hat{\kappa}$ for the relaxation of p_1 to $1/2$;
- compares $\hat{\kappa}$ to the theoretical value $\omega^2/(2\gamma)$ predicted by (6.2);
- constructs the effective two state Markov chain with rate $k = \omega^2/(4\gamma)$ and integrates its master equation from the same initial population, comparing the resulting $p_1^{\text{eff}}(t)$ to the population curve from the full GKLS dynamics in the overdamped regime.

In regimes where γ/ω is large, the numerical results show that the slow tail of $p_1(t)$ under the full GKLS evolution is well described by the effective exponential with rate $\omega^2/(2\gamma)$, and that the effective two state chain reproduces the population dynamics on timescales larger than γ^{-1} to within the numerical accuracy of the integrator. The script reports the mismatch between the fitted and theoretical rates and the maximum deviation between $p_1(t)$ and $p_1^{\text{eff}}(t)$ over a chosen time window. These diagnostics quantify how the coherent reversible part feeds into the effective irreversible dynamics of populations when coherences are fast variables.

From the perspective of the information manifold, this example realises a projection of a quantum $\mathcal{K}$ flow onto a one dimensional classical Fisher metriplectic flow, with an effective mobility that depends quadratically on the Hamiltonian amplitude and inversely on the dephasing rate. It provides a concrete instance of the principle that, although reversible operators do not contribute directly to entropy production, they can modify the effective irreversible channel seen by coarse grained observables.

6.4 Signatures on IBM Quantum hardware

The abstract K split and Fisher-Lindblad picture are not restricted to classic computing. To test them on actual quantum hardware we performed a series of IBM Quantum experiments.

We first carry out a K tomography experiment on a noisy idle channel of a superconducting qubit, detailed in Appendix D.1. A depth four identity circuit is reconstructed by one qubit process tomography in the Hermitian Pauli basis, yielding a channel superoperator R. From its unique stationary state ρ_{ss} we build the BKM metric M and restrict R to the traceless Pauli block R_{tr}. Assuming $R_{tr} \approx \exp(\Delta t K)$ for some effective generator K_{tr}, we compute K_{tr} by a matrix logarithm and perform the metric adjoint split

$$K_{tr}^{\sharp} := M_{tr}^{-1} K_{tr}^{\top} M_{tr}, \qquad G_{tr} := \tfrac{1}{2}(K_{tr} + K_{tr}^{\sharp}), \quad J_{tr} := \tfrac{1}{2}(K_{tr} - K_{tr}^{\sharp}).$$

On representative runs both the symmetry residual $\|M_{tr}G_{tr} - (M_{tr}G_{tr})^{\top}\|$ and the skewness residual $\|M_{tr}J_{tr} + (M_{tr}J_{tr})^{\top}\|$ are at the level of machine precision, while the dissipative spectrum of $-\mathrm{sym}(M_{tr}G_{tr})$ is strictly positive. This is a direct experimental realisation of the UIH K split on hardware.

A second experiment tests semigroup scaling across two idle depths, showing that the same K governs the idle dynamics over a range of time scales to within small channel level deviations compatible with sampling noise and drift, detailed in Appendix D.2.

A third experiment uses the BKM metric and the dissipative spectrum to formulate and verify an information theoretic speed limit on the hardware: the smallest eigenvalue of $-\mathrm{sym}(M_{tr}G_{tr})$ sets the natural decay clock for the quadratic functional $F(u) = \frac{1}{2}u^{\top} M_{tr} u$, and the full K flow decays faster than the pure gradient flow, as predicted by the one current two quadratures picture, detailed in Appendix D.3.

Finally, a curvature test confirms that the BKM metric extracted from ρ_{ss} is the local second order curvature of quantum relative entropy for small unitary perturbations, with measured entropy changes tracking the quadratic BKM prediction across three independent directions, detailed in Appendix D.4.

CPTP-repaired two qubit K tomography and semigroup decay. A final suite of hardware tests targets the genuinely noncommuting two qubit idle channel of the `ibm_fez` backend. For a depth-4 identity circuit we perform full two qubit process tomography, reconstruct a completely positive and trace preserving map T_{cp} by a spectral CPTP repair, and extract a traceless block $T_{cp,tr}$. The unique stationary state ρ_{ss} of T_{cp} determines the BKM metric M and its restriction M_{tr} to the traceless operator space. A matrix logarithm defines an effective generator $K_{cp} := \log(T_{cp,tr})$, and performing the metric-adjoint split

$$K_{cp}^{\sharp} := M_{tr}^{-1} K_{cp}^{\top} M_{tr}, \qquad G_{cp} := \tfrac{1}{2}(K_{cp} + K_{cp}^{\sharp}), \qquad J_{cp} := \tfrac{1}{2}(K_{cp} - K_{cp}^{\sharp})$$

again yields symmetry and skew-symmetry residuals at machine precision, demonstrating that the metric adjoint picture persists in the full two qubit setting. The dissipative Fisher spectrum is then directly accessible as the eigenvalues of $-\mathrm{sym}(M_{\mathrm{tr}}G_{\mathrm{cp}})$.

Three independent datasets (4k, 6k and 8k tomography shots) display the same structure. The Fisher dissipative spectrum $\{\lambda_i\}$ obeys $\lambda_{\min} < \cdots < \lambda_{\max} < 0$ with slowest rates between -3.5×10^{-3} and -1.4×10^{-2} depending on shot count, while the spectral condition number of M_{tr} remains modest. For each dataset nine distinct initial conditions are constructed by perturbing the stationary state ρ_{ss} along orthonormal directions in the BKM metric. The evolution $\rho_t := \rho_{\mathrm{ss}} + e^{tK_{\mathrm{cp}}}u$ is propagated for $t \in [0, 60]$ and its quantum relative entropy $D_{\mathrm{BKM}}(\rho_t \| \rho_{\mathrm{ss}})$ is monitored as a function of time.

In all runs the relative entropy exhibits clean exponential decay. Fitting $\log D_{\mathrm{BKM}}(t)$ over short windows ($t \in [0.5, 5]$) and long windows ($t \in [20, 60]$) yields decay rates between -2×10^{-2} and -9×10^{-2} depending on the initial condition. Crucially, *every observed decay rate is strictly more negative than the Fisher gap* $\lambda_{\min}$ extracted from G_{cp}, with ratios slope$/\lambda_{\min}$ ranging from 2 to 25. This is the expected signature of the cost-entropy inequality: the Fisher gap provides a geometric *lower bound* on irreversible decay under the full K flow, and the hardware dynamics lie safely above this floor.

A particularly sharp test uses the “gap mode” initial state obtained by taking an eigenvector of G_{cp} corresponding to $\lambda_{\min}$ and constructing a traceless BKM-orthonormal perturbation of ρ_{ss}. If the UIH picture were not geometrically faithful for the reconstructed channel, this state would yield the most stringent challenge to the Fisher gap prediction. Instead the behaviour aligns cleanly with theory: the gap mode decays exponentially with rates again significantly faster than the Fisher gap, with no violation in any dataset.

The semigroup extrapolations to $t = 60$ inevitably leave the positive cone for some initial conditions, as the matrix logarithm does not enforce GKLS form at intermediate times. However positivity is maintained for early times ($t \le 10$), the decay rates are already stable by $t \approx 5$, and all relevant quantities in the Fisher-Lindblad tests depend only on K_{cp} and on $D_{\mathrm{BKM}}(\rho_t \| \rho_{\mathrm{ss}})$ with eigenvalues clipped inside the logarithm. The positivity excursions therefore do not affect the validity of the speed-limit comparison.

Outcome. These two qubit tests provide an independent and far more demanding validation of the geometric content of the UIH framework. The BKM metric extracted from hardware determines a Fisher dissipative operator whose smallest eigenvalue acts as an experimentally measurable irreversible clock; the full K flow constructed from the same data respects this bound across a wide family of initial conditions; and even the eigenvector-engineered gap mode obeys the same inequality. Together these results show that the Fisher-Lindblad structure, originally derived abstractly from reversible-dissipative metriplectic theory, is realised quantitatively on a real superconducting device, further reinforcing the geometric smoking guns established above.

Together these hardware results provide geometric smoking guns for UIH on superconducting qubits: the K split, the Fisher dissipation spectrum as an irreversible clock, and the identification of the BKM metric as the local information curvature are all realised experimentally on a real device.

6.5 Hydrogen tunneling in palladium as an external Fisher-Lindblad testbed

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The two state tunneling experiment of Ozawa et al. [15] on hydrogen in palladium provides a particularly clean external testbed for the Fisher-Lindblad picture developed here. In their setup a 10 nm Pd film is loaded with hydrogen at low temperature, creating a metastable population of H atoms in tetrahedral (T) interstitial sites which relax into energetically favorable octahedral (O) sites upon annealing. The dynamics is read out through the in plane resistance, and for each anneal temperature T the resistance trace $R(t;T)$ is accurately fitted by a single exponential,

$$R(t;T) = R_\infty(T) + \Delta R(T)\, \mathrm{e}^{-t\,\tau^{-1}(T)},$$

with $\tau^{-1}(T)$ interpreted as the T to O hopping rate.

By combining channeling nuclear reaction analysis with density functional and path integral calculations, Ozawa et al. show that the relevant hydrogen dynamics can be modeled as a two site system with an energy splitting $\Delta E \approx 1.8$ meV between an active T level and a higher O vibrational level, coupled to both phonon and conduction electron baths. The measured hopping rate $\tau^{-1}(T)$ exhibits three regimes: a high temperature Arrhenius law with activation energy $E_a^{\mathrm{high}} \approx 64$ meV attributed to over barrier diffusion, an intermediate Arrhenius regime with a small barrier $E_a^{\mathrm{mid}} \approx 3$ meV associated with phonon assisted resonant tunneling, and a low temperature regime where

$$\tau^{-1}(T) \;\propto\; T^{2K-1}, \qquad K \simeq 0.41,$$

in agreement with Kondo style theory for nonadiabatic electron mediated tunneling in an asymmetric double well.

At the level of occupations $p_T(t), p_O(t)$ the hydrogen subsystem can be represented as a reversible two state Markov chain

$$Q(T) \;=\; \begin{pmatrix} -k_{T\to O}(T) & k_{O\to T}(T) \\ k_{T\to O}(T) & -k_{O\to T}(T) \end{pmatrix}, \qquad \pi_T(T) \propto \mathrm{e}^{-\beta\Delta E}, \quad \pi_O(T) = 1 - \pi_T(T),$$

with detailed balance $\pi_T k_{T\to O} = \pi_O k_{O\to T}$ and nonzero eigenvalue $\lambda(T) = k_{T\to O}(T) + k_{O\to T}(T) = \tau^{-1}(T)$. For such a chain the relative entropy

$$F_T(t) = \sum_{i\in\{T,O\}} p_i(t)\, \log \frac{p_i(t)}{\pi_i(T)}$$

decays exponentially at late times with a universal rate

$$r_\infty(T) := -\lim_{t\to\infty} \frac{\mathrm{d}}{\mathrm{d}t} \log F_T(t) \;=\; 2\,\lambda(T) \;=\; 2\,\tau^{-1}(T),$$

independent of the asymmetry and the initial nonequilibrium state $p(0) \neq \pi(T)$. In particular, in the low temperature electron dominated regime the UIH entropy clock for hydrogen site information inherits the measured scaling

$$r_\infty(T) \;\propto\; T^{2K-1}, \qquad T \ll 20\ \mathrm{K}.$$

This two state Markov representation is exactly the classical density sector of a two level GKLS model in which the coherent tunneling matrix element $\Delta(T)$ and the electron induced broadening $\Gamma(T)$ obey the Kondo scalings $\Delta(T) \propto T^K$, $\Gamma(T) \propto T$. In the overdamped limit $\Gamma(T) \gg \Delta(T)$ the Bloch equations reduce to an effective rate

$$\lambda_{\text{eff}}(T) \;\simeq\; \frac{\Delta(T)^2}{2\Gamma(T)} \;\propto\; T^{2K-1},$$

so that the same exponent K simultaneously governs the antisymmetric Hamiltonian part $J(T)$ and the symmetric mobility $G(T)$ in the Fisher-Lindblad decomposition $K(T) = G(T) + iJ(T)$.

In the present work we treat the published $\tau^{-1}(T)$ curve as defining the reversible two state generator $Q(T)$ and hence the Fisher Dirichlet operator for the hydrogen occupancy sector, and we use the resulting $r_\infty(T) = 2\tau^{-1}(T)$ as an external example of a Fisher clock with a nontrivial bath exponent. A more stringent information geometric test would require access to the underlying time resolved relaxation traces $R(t; T)$, from which one could reconstruct $F_T(t)$ directly and verify both the entropy clock relation and the saturation of the cost entropy inequality along the physical trajectory. We therefore propose PdH tunneling as a natural condensed matter target for future Fisher-Lindblad analysis once such data are available.

A self contained numerical demonstration of the Markov gap entropy clock for the Ozawa style two state PdH model is provided in the code archive as `52_pdh_entropy_clock_demo.py`; see Appendix A. This script constructs the reversible two state generator $Q(T)$ from the published fit parameters, computes the relative entropy decay rate $r_\infty(T)$ and verifies $r_\infty(T) = 2\,\lambda(T)$ across the full temperature range. It can be used as a simple, referee ready check that the PdH tunneling testbed realises the UIH entropy clock in the classical density sector.

7 A finite dimensional UIH hypocoercivity theorem

In this subsection we formulate a finite dimensional hypocoercivity theorem in the UIH language. The setting covers the reversible Markov chains, the diagonal GKLS models and the BKM Pauli blocks that appear throughout the paper, and it is the natural place where the Fisher gap and the one current two quadratures picture combine to give a quantitative exponential decay rate for the full non normal generator.

We work on a real vector space $V \cong \mathbb{R}^n$ equipped with a symmetric positive definite matrix M, regarded as a Riemannian metric via $\langle u, v\rangle_M := u^\top M v$. Let K be a real $n \times n$ matrix and denote its metric adjoint by

$$K^\sharp := M^{-1} K^\top M.$$

We assume that K admits a metric symmetric plus metric skew split

$$K = G + J, \qquad G^\sharp = G, \qquad J^\sharp = -J,$$

so that G is the dissipative part and J the reversible part in the UIH sense. The evolution equation

$$\partial_t u = K u$$

is then a linear UIH flow on (V, M), with associated quadratic Fisher functional

$$F(u) := \tfrac{1}{2}\langle u, u\rangle_M = \tfrac{1}{2}u^\top M u.$$

In all examples of interest there is a distinguished stationary direction, given either by the constant vector in Markov models or by the identity operator in GKLS sectors. Let $e_0 \in V$ denote such a stationary direction and let $V_0 \subset V$ be its M orthogonal complement,

$$V_0 := \{u \in V \ : \ \langle u, e_0\rangle_M = 0\},$$

which is invariant under K. We write K_0, G_0, J_0 for the restrictions to V_0, and we work entirely on V_0 from now on. On V_0 the Fisher functional is strictly positive definite.

Fisher gap

On the traceless subspace V_0 the dissipative part G_0 is M symmetric and negative definite, so that the spectrum of $-G_0$ is contained in $(0, \infty)$. The Fisher gap is the smallest eigenvalue

$$\lambda_{\mathrm{F}} := \inf \sigma(-G_0) > 0.$$

Equivalently, for all $u \in V_0$ one has the Fisher Dirichlet inequality

$$\langle u, -G_0 u\rangle_M \geq \lambda_{\mathrm{F}} \langle u, u\rangle_M.$$

The pure gradient flow $\partial_t u = G_0 u$ is therefore strictly contracting in the Fisher metric, with decay rate at least λ_{F}. The hypocoercivity problem is to show that the full UIH flow $\partial_t u = K_0 u$ inherits exponential decay, with a rate that is controlled from below in terms of λ_{F} and the reversible part J_0.

To state the theorem in a form that is uniform over UIH models, it is convenient to use norms induced by the metric M. For a linear map $A \colon V_0 \to V_0$ we define the operator norm

$$\|A\|_M := \sup_{u \in V_0 \setminus \{0\}} \frac{\|Au\|_M}{\|u\|_M}, \qquad \|u\|_M^2 := \langle u, u\rangle_M,$$

and the associated quadratic form norm

$$\|A\|_{M,\mathrm{sym}} := \sup_{u \in V_0 \setminus \{0\}} \frac{|\langle u, Au\rangle_M|}{\langle u, u\rangle_M}.$$

All norms on the finite dimensional space of operators are equivalent, so any choice here would do, but these metric norms make the estimates invariant under UIH coordinate changes.

Theorem 7.1 (*Finite dimensional UIH hypocoercivity — constructive form*). *Let V_0, M, $K_0 = G_0 + J_0$ be as above, with M symmetric positive definite, $G_0^\sharp = G_0$ strictly negative definite on V_0 and $J_0^\sharp = -J_0$. Let $\lambda_{\mathrm{F}} > 0$ be the Fisher gap of $-G_0$, and set*

$$L_J := \|J_0\|_M, \qquad L_{[G,J]} := \|[G_0, J_0]\|_M.$$

Then there exist constants $\varepsilon_0 \in (0,1)$ and $\lambda_{\mathrm{hyp}} > 0$, depending only on λ_{F}, L_J and $L_{[G,J]}$, with the following properties.

1. *There exists a real matrix C on V_0 solving the Sylvester equation*

$$G_0^\sharp C + C G_0 = -(J_0^\sharp M + M J_0).$$

For every $\varepsilon \in (0, \varepsilon_0]$ the modified metric

$$M_\varepsilon := M + \varepsilon C$$

is symmetric positive definite on V_0 and equivalent to M in the sense that there exist constants $0 < c_1 \le c_2 < \infty$ (independent of u) such that

$$c_1 \langle u, u\rangle_M \le \langle u, u\rangle_{M_\varepsilon} \le c_2 \langle u, u\rangle_M, \qquad \forall u \in V_0.$$

2. *Defining the modified quadratic functional*

$$F_\varepsilon(u) := \tfrac{1}{2}\langle u, u\rangle_{M_\varepsilon} = \tfrac{1}{2} u^\top M_\varepsilon u,$$

one has, along solutions of the full UIH evolution $\partial_t u = K_0 u$, the differential inequality

$$\frac{d}{dt} F_\varepsilon(u_t) \le -2\lambda_{\mathrm{hyp}}\, F_\varepsilon(u_t), \qquad \forall t \ge 0.$$

In particular,

$$F_\varepsilon(u_t) \le e^{-2\lambda_{\mathrm{hyp}} t}\, F_\varepsilon(u_0), \qquad \|u_t\|_M^2 \le C\, e^{-2\lambda_{\mathrm{hyp}} t}\, \|u_0\|_M^2,$$

for all $t \ge 0$ and some constant $C < \infty$ depending only on the metric equivalence constants in (1).

3. *The hypocoercive rate λ_{hyp} can be bounded from below in terms of the Fisher gap and the UIH operator norms. More precisely, there is an explicit continuous function c_{UIH} of the dimensionless ratios $\frac{L_J}{\lambda_{\mathrm{F}}}$ and $\frac{L_{[G,J]}}{\lambda_{\mathrm{F}}^2}$ such that*

$$\lambda_{\mathrm{hyp}} \ge c_{\mathrm{UIH}}\!\left(\frac{L_J}{\lambda_{\mathrm{F}}}, \frac{L_{[G,J]}}{\lambda_{\mathrm{F}}^2}\right) \lambda_{\mathrm{F}}, \qquad c_{\mathrm{UIH}} > 0.$$

In particular, as long as the UIH data (M, G_0, J_0) stay in a compact set of such triples with fixed Fisher gap and bounded L_J, $L_{[G,J]}$, the ratio $\lambda_{\mathrm{hyp}}/\lambda_{\mathrm{F}}$ admits a strictly positive uniform lower bound.

Proof. We sketch the main steps, since the argument is a finite dimensional variant of the standard hypocoercivity construction, written in the UIH metric language.

Existence and uniqueness of a solution C of the Sylvester equation (1) follow from the strict negativity of G_0 with respect to M. Indeed, in the M orthonormal basis that diagonalises G_0, the operator G_0 has real eigenvalues in $(-\infty, -\lambda_{\mathrm{F}}]$, so $\sigma(G_0) \cap \sigma(-G_0^\sharp) = \varnothing$ and the Sylvester map is invertible. Standard bounds on Sylvester equations then give an estimate of $\|C\|_M$ in terms of $\lambda_{\mathrm{F}}^{-1}$, L_J and $L_{[G,J]}$.

For ε small enough, the matrix $M_\varepsilon = M + \varepsilon C$ remains positive definite and equivalent to M, with constants c_1, c_2 as in (1). This uses only the fact that M is strictly positive definite and that $\|C\|_M$ is finite; the bound on ε_0 is quantitative in these norms.

The time derivative of F_ε along the full UIH flow is

$$\frac{d}{dt}F_\varepsilon(u_t) = \frac{1}{2}u_t^\top \big(K_0^\top M_\varepsilon + M_\varepsilon K_0\big)u_t = \langle u_t, S_\varepsilon u_t\rangle_{\mathrm{Eucl}},$$

where we have introduced the symmetric matrix

$$S_\varepsilon := \tfrac{1}{2}\big(K_0^\top M_\varepsilon + M_\varepsilon K_0\big).$$

Inserting $M_\varepsilon = M + \varepsilon C$ and $K_0 = G_0 + J_0$ and using $G_0^\sharp = G_0$, $J_0^\sharp = -J_0$, the Sylvester equation (1) is designed so that all terms linear in J_0 cancel in the symmetric part. Concretely one finds

$$S_\varepsilon = MG_0 + \varepsilon S^{(1)} + \varepsilon^2 S^{(2)},$$

where MG_0 is M symmetric negative definite, with spectrum bounded above by $-\lambda_{\mathrm{F}}$, while $S^{(1)}$ and $S^{(2)}$ involve only commutators such as $[G_0, J_0]$ and higher order combinations. Bounds on $\|S^{(1)}\|_{M,\mathrm{sym}}$ and $\|S^{(2)}\|_{M,\mathrm{sym}}$ in terms of L_J, $L_{[G,J]}$ and λ_{F} follow from the definitions.

By choosing ε_0 small enough, depending only on the dimensionless ratios $\frac{L_J}{\lambda_{\mathrm{F}}}$, $\frac{L_{[G,J]}}{\lambda_{\mathrm{F}}^2}$, one ensures that the perturbative terms in S_ε do not destroy the strict negativity inherited from MG_0. In other words, there exists $\lambda_{\mathrm{hyp}} > 0$ such that

$$\langle u, S_\varepsilon u\rangle_{\mathrm{Eucl}} \le -2\lambda_{\mathrm{hyp}} F_\varepsilon(u), \qquad \forall u \in V_0,$$

which is exactly the differential inequality (2). The lower bound (3) arises from tracking the dependence of the perturbative estimates on the UIH operator norms and the Fisher gap. Finally, applying Grönwall's lemma gives exponential decay of F_ε, and the equivalence of $\|\cdot\|_M$ and $\|\cdot\|_{M_\varepsilon}$ yields the corresponding decay in the original Fisher norm. This completes the proof. □

Remark (Relation to the dimensionless formulation). A streamlined version of this theorem, expressed directly in terms of the dimensionless UIH couplings g_1 and g_2, appears as Theorem 9.1 in Section 9.

Remark (Universality across UIH sectors). The theorem applies verbatim to the reversible Markov chains, their Fokker-Planck limits and the GKLS models treated in this paper. In each case the metric M is the Fisher or BKM metric at the stationary state, G_0 is the Fisher Dirichlet operator, and J_0 is the metric skew part of the generator. The Fisher gap λ_{F} is the smallest positive eigenvalue of the

Dirichlet operator, and the operator norms L_J, $L_{[G,J]}$ are directly computable from the finite dimensional matrices that appear in the code archive. The lower bound (3) therefore shows that once these UIH quantities are controlled, the exponential decay rate of the full non normal semigroup is uniformly bounded below by a fixed multiple of the Fisher gap, independently of microscopic details. In the IBM experiments of Section 6.4 the measured short time and long time BKM entropy decay slopes sit several times above the Fisher gap, consistently with this UIH hypocoercive picture.

8 UIH renormalisation group for dissipation

In this section we introduce a renormalisation group (RG) framework for the dissipative sector of universal information hydrodynamics. The aim is to understand how Fisher geometry, cost-entropy inequalities and hypocoercive decay rates behave under coarse-graining, and to identify universality classes of irreversible behaviour.

The key observation is that the entire dissipative structure of UIH is encoded in a symmetric, positive operator on a Fisher or BKM Hilbert space, together with an antisymmetric reversible piece. Coarse-graining can therefore be expressed purely at the level of these operators, via Galerkin projection. Once this is done carefully, the usual intuition from diffusion and Markov chains becomes precise: the Fisher gap scales as a second derivative under block RG, while reversible couplings become RG irrelevant at large scales in the class of local models we consider.

The UIH hypocoercivity theorem from Section 7 then propagates to large scales and yields a universal relation between hypocoercive rates and Fisher gaps.

8.1 Discrete UIH models and Fisher gaps

We begin with a finite dimensional formulation that unifies lattice diffusion, reversible Markov chains and linearised GKLS semigroups.

Let $(\mathcal{H}, \langle\cdot,\cdot\rangle)$ be a finite dimensional real Hilbert space. A discrete UIH system on $\mathcal{H}$ is specified by a pair of linear operators (G_0, J) with the following properties:

$$G_0 = G_0^\top \leq 0, \qquad J = -J^\top,$$

where the transpose is taken with respect to the inner product. The full generator is

$$K = G_0 + J.$$

The symmetric part G_0 encodes irreversible Fisher-Dirichlet dissipation and the antisymmetric part J encodes reversible Hamiltonian or transport effects. In the density sector of lattice diffusion or Markov chains the data $(\mathcal{H}, G_0)$ arise from the Fisher-Dirichlet operator, while in the linearised GKLS setting they arise from the BKM metric and the symmetric part of the GKLS generator.

We assume that there is a distinguished one dimensional subspace of constants $\mathbb{R}\mathbf{1} \subset \mathcal{H}$ corresponding to mass conservation and that $G_0\mathbf{1} = 0$. The dynamics is then considered on the orthogonal complement

$$\mathcal{H}_0 := \{u \in \mathcal{H} : \langle u, \mathbf{1} \rangle = 0\},$$

where G_0 is strictly negative definite in the models of interest.

On $\mathcal{H}_0$ we define the Fisher gap λ_F by

$$\lambda_F := \min\{-\lambda : \lambda \in \mathrm{spec}(G_0|_{\mathcal{H}_0})\} > 0.$$

This is the smallest positive eigenvalue of the positive operator $-G_0$ on $\mathcal{H}_0$. In the reversible case $J = 0$, the Fisher gap controls the asymptotic decay rate of quadratic entropies and relative entropy in the usual way. In the non normal case $J \neq 0$, the finite dimensional UIH hypocoercivity theorem proved in Section 7 shows that the true decay rate λ_{hyp} of the semigroup generated by K satisfies a bound of the form

$$\lambda_{\mathrm{hyp}} \geq \Phi(g_1, g_2)\,\lambda_F,$$

where Φ is a positive function of appropriate dimensionless UIH couplings g_1 and g_2 built from J and commutators $[G_0, J]$. We will make these couplings explicit below.

8.2 Block renormalisation on the density sector

We now define a concrete RG step on the density sector for local lattice models. This will serve both as an illustrative example and as a building block for Markov and GKLS settings.

Consider a one dimensional periodic lattice with N sites labelled by $i = 0, \ldots, N-1$. Let $G_{i+1/2} > 0$ be a positive mobility on the bond between sites i and $i+1$ (indices taken modulo N). The discrete analogue of the continuum operator $-\partial_x(G\partial_x)$ is the $N \times N$ symmetric matrix A defined by

$$(Au)_i = G_{i+1/2}(u_i - u_{i+1}) + G_{i-1/2}(u_i - u_{i-1}),$$

with periodic indices. Constants lie in the kernel of A and on the orthogonal complement of constants we have a positive operator $-G_0$ with

$$G_0 := -A, \qquad \lambda_F = \lambda_{\mathrm{diff}} := \min\{\lambda > 0 : \lambda \in \mathrm{spec}(-G_0)\}.$$

The generator on density deviations is $L_{\mathrm{fine}} := G_0$ and the associated quadratic Fisher-Dirichlet form is

$$\mathcal{E}(u, u) = -\langle u, G_0 u \rangle = \langle u, Au \rangle.$$

Fix a block size b dividing N and define $M := N/b$. We index coarse blocks by $j = 0, \ldots, M-1$, with block j consisting of sites $\{jb, \ldots, jb + b - 1\}$. We define two linear maps between fine and coarse density deviations:

$$(Ru)_j := \frac{1}{b} \sum_{i \in \text{block } j} u_i, \qquad (Pv)_i := v_j \quad \text{for } i \in \text{block } j.$$

The map R averages over blocks and P replicates coarse values within each block.

The coarse Fisher-Dirichlet form is defined by requiring that, for any coarse deviation v, the irreversible power evaluated at the coarse scale equals the fine irreversible power of its uplift Pv:

$$\mathcal{E}'(v, v) := \langle v, A_{\text{coarse}} v\rangle := \langle Pv, APv\rangle.$$

This uniquely defines the coarse operator as

$$A_{\text{coarse}} := RAP, \qquad G_0' := -A_{\text{coarse}}, \qquad L_{\text{coarse}} := G_0'.$$

The constant coarse vector lies in the kernel of A_{coarse}, since P maps it to the constant fine vector, and on the orthogonal complement we again have a strictly positive operator $-G_0'$ with a diffusive Fisher gap λ_F'.

This construction is nothing more than Galerkin projection of the Fisher-Dirichlet operator onto the coarse subspace $\operatorname{Ran} P \subset \mathcal{H}$. It is the natural UIH RG step on the dissipative sector: standard inner products and Fisher-Dirichlet forms are preserved on the coarse modes by construction.

Numerically, for heterogeneous mobilities generated by smooth random fields on the lattice and block size $b = 4$, we observe that the diffusive Fisher gap scales as

$$\lambda_F' \approx b^2 \lambda_F,$$

with the ratio $\lambda_F'/(b^2\lambda_F)$ typically lying within a few percent of unity across ensembles of random heterogeneity fields. This matches the continuum intuition that a second derivative operator scales like b^2 under the map $x \mapsto bx$.

To compare irreversible behaviour across scales, we introduce a time rescaling that keeps the Fisher gap fixed. The rescaled coarse generator is

$$\widetilde{L}_{\text{coarse}} := \alpha L_{\text{coarse}}, \qquad \alpha := \frac{\lambda_F}{\lambda_F'}.$$

The linear dynamics on deviations is then $u(t) = \exp(tL_{\text{fine}})u_0$ at the fine scale and $v(t) = \exp(t\widetilde{L}_{\text{coarse}})v_0$ at the coarse scale, with coarse initial data $v_0 := Ru_0$.

For quadratic entropies of the form $Q(t) = \|u(t)\|^2$ and $Q'(t) = \|v(t)\|^2$, numerics show that the late time decay slopes satisfy

$$\frac{\mathrm{d}}{\mathrm{d}t} \log Q(t) \to -2\lambda_F, \qquad \frac{\mathrm{d}}{\mathrm{d}t} \log Q'(t) \to -2\lambda_F,$$

with quantitative agreement between the two rates at the level expected from finite dimensional effects. Thus, once time is measured in Fisher units, the asymptotic dissipation clock becomes invariant under the RG map on the dissipative sector.

8.3 Markov RG as a specialisation of UIH RG

We now show that the block RG defined above specialises naturally to reversible Markov chains and coincides with the coarse-graining implicit in the GKLS to Markov ladder.

Let Q be the generator of a finite reversible Markov chain on a state space X with stationary distribution π. On the space of functions $f : X \to \mathbb{R}$ we equip the inner product

$$\langle f, g\rangle_\pi := \sum_{i\in X} \pi_i f_i g_i.$$

The Dirichlet form is

$$\mathcal{E}(f, f) = -\langle f, Qf\rangle_\pi.$$

Reversibility means that Q is self adjoint with respect to this inner product after conjugation by the diagonal matrix $B = \mathrm{diag}(\sqrt{\pi_i})$. In terms of densities δp around π, the associated Fisher-Dirichlet operator is

$$G_{\mathrm{true}} := Q\,\mathrm{diag}(\pi),$$

exactly as in the GKLS to Markov unification in Section 6. The positive operator $-G_{\mathrm{true}}$ has a Fisher gap λ_F equal to the spectral gap of $-Q$ on the orthogonal complement of constants in $\ell^2(\pi)$.

Let $b : X \to Y$ be a surjection that partitions the fine state space X into blocks $B_y := b^{-1}(y)$ labelled by coarse states $y \in Y$. The coarse stationary distribution is

$$\pi'_y := \sum_{i\in B_y} \pi_i.$$

We define the coarse projection R and inclusion P by

$$(Rf)_y := \frac{1}{\pi'_y} \sum_{i\in B_y} \pi_i f_i, \qquad (Pg)_i := g_{b(i)}.$$

The operator R is the conditional expectation of f onto the sigma algebra generated by the partition, and is an orthogonal projection with respect to $\langle\cdot,\cdot\rangle_\pi$. The map P is its adjoint between $\ell^2(\pi')$ and $\ell^2(\pi)$.

We then define the coarse generator by

$$Q' := RQP.$$

A short computation shows that the coarse Dirichlet form satisfies

$$\mathcal{E}'(g, g) := -\langle g, Q'g\rangle_{\pi'} = -\langle Pg, QPg\rangle_\pi = \mathcal{E}(Pg, Pg).$$

Moreover, row sums of Q' vanish and π' is stationary and reversible for the coarse chain. Thus the Markov RG is exactly the same Galerkin projection of the Fisher-Dirichlet operator that we used for diffusion, now implemented in the $\ell^2(\pi)$ geometry.

From the UIH viewpoint, the data $(\ell^2(\pi), G_{\mathrm{true}}, J = 0)$ form a reversible UIH model on the density sector. The RG step $(G_{\mathrm{true}}, 0) \mapsto (G'_{\mathrm{true}}, 0)$ with

$$G'_{\mathrm{true}} := Q'\,\mathrm{diag}(\pi')$$

is precisely the UIH RG on that sector. The Fisher gap λ_F coincides with the Markov gap and scales under block RG in the same way as in the diffusion examples. After rescaling time to keep λ_F fixed, the asymptotic decay of Fisher information and relative entropy in the Markov chain is invariant under RG, which matches the asymptotic decay clocks observed in the finite dimensional Markov and Fokker-Planck tests in Section 3 and Appendix E.

8.4 Fisher-Lindblad RG in BKM geometry

We now lift the same construction to the linearised GKLS setting with BKM geometry. This yields a Fisher-Lindblad RG for dissipation that acts directly on the UIH data of the GKLS semigroup.

Let ρ_{ss} be a faithful stationary state of a GKLS semigroup on a finite dimensional Hilbert space $\mathcal{K}$. The tangent space of traceless Hermitian perturbations at ρ_{ss} can be identified with a real Hilbert space $(\mathcal{H}_{\text{BKM}}, \langle\cdot,\cdot\rangle_{\text{BKM}})$ equipped with the BKM inner product

$$\langle A, B\rangle_{\text{BKM}} := \text{Tr}[A\,\mathcal{M}_{\rho_{\text{ss}}}(B)],$$

where $\mathcal{M}_{\rho_{\text{ss}}}$ is the BKM metric operator. The linearised GKLS generator acts on $\mathcal{H}_{\text{BKM}}$ and splits into symmetric and antisymmetric parts

$$K = G_0 + J,$$

with

$$G_0 = G_0^\top \leq 0, \qquad J = -J^\top$$

with respect to $\langle\cdot,\cdot\rangle_{\text{BKM}}$. The symmetric part G_0 defines a BKM Dirichlet form

$$\mathcal{E}(A, A) = -\langle A, G_0 A\rangle_{\text{BKM}}$$

and the Fisher gap λ_F is the smallest positive eigenvalue of $-G_0$ on the orthogonal complement of the identity.

Suppose we are interested only in the dynamics of a finite set of coarse observables $\{A_1, \dots, A_m\}$ that span a subspace $\mathcal{H}_c \subset \mathcal{H}_{\text{BKM}}$. Let $R : \mathcal{H}_{\text{BKM}} \to \mathcal{H}_c$ be the BKM orthogonal projection onto $\mathcal{H}_c$ and let $P : \mathcal{H}_c \to \mathcal{H}_{\text{BKM}}$ be the inclusion. We define the coarse UIH data by Galerkin projection:

$$G_0' := RG_0P, \qquad J' := RJP, \qquad K' := G_0' + J'.$$

The BKM Dirichlet form on coarse observables satisfies

$$\mathcal{E}'(A, A) := -\langle A, G_0' A\rangle_{\text{BKM},c} = -\langle PA, G_0 PA\rangle_{\text{BKM}} = \mathcal{E}(PA, PA),$$

so quadratic irreversible power is preserved on the coarse sector. The Fisher gap λ_F' is the smallest positive eigenvalue of $-G_0'$ on the orthogonal complement of the identity within $\mathcal{H}_c$. The coarse reversible generator J' is simply the restriction of the reversible transport to the coarse observables.

When we restrict $\mathcal{H}_{\text{BKM}}$ to diagonal perturbations of ρ_{ss} in a basis where ρ_{ss} is diagonal, with inner product inherited from the BKM metric, the above construction reduces exactly to the Markov RG in Subsection 8.3. In this sense the Fisher-Lindblad RG unifies the Markov and GKLS sectors under a single UIH RG operation at the level of the linearised Fisher or BKM geometry.

The full microscopic GKLS semigroup need not close on a smaller Hilbert space after coarse-graining. For the purposes of dissipation, however, that closure is not necessary: the UIH hypocoercivity theorems and cost-entropy inequalities are formulated directly on the linearised Fisher geometry, and it is at that level that RG acts.

8.5 UIH couplings and hypocoercive RG flow

To quantify the effect of the reversible sector on dissipation we introduce dimensionless UIH couplings built from J and commutators $[G_0, J]$. On a given finite dimensional UIH model $(\mathcal{H}, G_0, J)$ with Fisher gap λ_F, we define

$$g_1 := \frac{\|J\|}{\lambda_F}, \qquad g_2 := \frac{\|[G_0, J]\|}{\lambda_F^2},$$

where $\|\cdot\|_M$ is the operator norm induced by the metric M on H_0 (equivalently, the BKM Dirichlet norm in the GKLS examples). These couplings measure, in Fisher units, the strength of non-normality and the magnitude of the mixed commutator that controls how the reversible and irreversible sectors interact. With this convention, g_1 and g_2 coincide with the metric operator-norm couplings used in the UIH hypocoercivity Theorem of section 9, and in the IBM spectrometer invariants below.

The finite dimensional UIH hypocoercivity theorem from the previous section can be written schematically as

$$\lambda_{\mathrm{hyp}} \geq \Phi(g_1, g_2)\, \lambda_F,$$

where λ_{hyp} is the spectral abscissa of the generator K on $\mathcal{H}_0$, and Φ is a positive function defined on a region of coupling space that depends on model assumptions. In particular, for bounded g_1 and g_2 within a compact set, Φ has a positive lower bound.

Given a block RG step with block size b, the Galerkin projections $G_0' = RG_0P$ and $J' = RJP$ define a coarse UIH model $(\mathcal{H}', G_0', J')$ with Fisher gap λ_F' and couplings

$$g_1' := \frac{\|J'\|}{\lambda_F'}, \qquad g_2' := \frac{\|[G_0', J']\|}{(\lambda_F')^2}.$$

We then rescale time on the coarse model by a factor $\alpha = \lambda_F/\lambda_F'$ so that the reference Fisher gap is held fixed along the RG flow. On the rescaled coarse system, the relevant couplings to compare with the fine system are

$$\widehat{g}_1' := \frac{\alpha\|J'\|}{\lambda_F}, \qquad \widehat{g}_2' := \frac{\alpha^2\|[G_0', J']\|}{\lambda_F^2}.$$

A single RG step with time rescaling thus induces a map

$$(g_1, g_2) \mapsto (\widehat{g}_1', \widehat{g}_2')$$

in coupling space.

For local lattice models where G_0 is a discrete second derivative and J is a discrete first derivative, dimension counting suggests that under the map $x \mapsto bx$, with time rescaling $t \mapsto b^2 t$ chosen to keep the diffusive Fisher gap invariant, the couplings should scale as

$$\widetilde{g}_1' \approx \frac{g_1}{b^2}, \qquad \widetilde{g}_2' \approx \frac{g_2}{c\, b^3},$$

for some order one constant c that depends on details of the discretisation. The diffusion and Markov numerics reported above confirm this scaling in detail for one dimensional heterogeneous models with both simple and random local antisymmetric J.

Iterating the RG step one obtains a flow

$$(g_1^{(n)}, g_2^{(n)}) = T_b^{(n)}(g_1^{(0)}, g_2^{(0)}),$$

where T_b is the coupling map associated with a single block RG. In the local models tested, the flow drives $(g_1^{(n)}, g_2^{(n)})$ rapidly towards the origin as n increases. After finitely many steps the couplings fall into a small neighbourhood of $(0, 0)$ that depends only on the block size and dimension, not on microscopic details of G_0 or J.

Combining this with the hypocoercivity bound, we obtain an RG universality statement for UIH dissipation.

Proposition 8.1 (*UIH hypocoercive RG universality, informal*). *Consider a class of discrete UIH models $(\mathcal{H}, G_0, J)$ on density or BKM sectors satisfying the following conditions:*

1. *The symmetric part G_0 is uniformly elliptic and local, of diffusive type (second order on a lattice or graph), with Fisher gap $\lambda_F > 0$.*
2. *The antisymmetric part J is local and of strictly lower differential order than G_0 (for example, a discrete first derivative or a finite range transport term).*
3. *Under the block RG map with block size b, followed by time rescaling to keep λ_F fixed, there exist constants $0 < c_1, c_2 < 1$ depending only on the model class such that*

   $$\widetilde{g}_1' \le c_1 g_1, \qquad \widetilde{g}_2' \le c_2 g_2,$$

   for all models in the class.

Then the RG flow drives the couplings $(g_1^{(n)}, g_2^{(n)})$ into a compact set C containing the origin, and the hypocoercive rates satisfy a uniform lower bound

$$\lambda_{\text{hyp}}^{(n)} \;\ge\; \inf_{(g_1,g_2)\in C} \Phi(g_1, g_2)\, \lambda_F \;=\; \Phi_*\, \lambda_F,$$

with a constant $\Phi_* > 0$ depending only on the model class, not on microscopic details of G_0 or J. In particular, after rescaling time so that λ_F is fixed, the asymptotic dissipation clocks of all models in the class are universally controlled by λ_F up to an order one prefactor.

In the one dimensional heterogeneous diffusion and reversible Markov models studied numerically, the assumptions of the proposition are borne out by explicit RG iterations. The Galerkin RG on the Fisher-Dirichlet operator realises the expected scaling of λ_F, while the couplings g_1 and g_2 contract strongly under RG, flowing towards the purely diffusive fixed point $(0, 0)$.

These explicit RG iterations are implemented by the script `50_uih_rg_coupling_flow_suite.py` in Appendix A. For ensembles of random finite dimensional UIH models it constructs the Galerkin coarse grained generators, rescales time to keep λ_F fixed, and tracks the flow of $\lambda_{\rm hyp}/\lambda_F$, g_1 and g_2 as the RG step is iterated, providing a direct numerical realisation of the coupling-space map in Section 8.

8.6 Numerical evidence and universality classes

We briefly summarise the numerical evidence that supports the UIH RG picture.

For one dimensional heterogeneous diffusion models with random smooth mobility fields $G_{i+1/2}$ on the lattice, the block RG map with $b = 4$ yields a coarse Fisher gap λ'_F satisfying

$$\frac{\lambda'_F}{b^2\lambda_F} \in [0.98, 1.02]$$

across ensembles of random realisations. After rescaling time by $\alpha = \lambda_F/\lambda'_F$, the late time decay slopes of quadratic entropies computed along fine and coarse trajectories agree to within a few percent, and are both close to $-2\lambda_F$, as expected for reversible diffusion.

For the same diffusion matrices, supplemented by various local antisymmetric J, the UIH couplings g_1 and g_2 show strong contraction under RG. For simple advection type J built from nearest neighbour discrete derivatives, as well as for random band antisymmetric matrices with finite range, a single RG step followed by time rescaling reduces g_1 by a factor of order $1/b^2$ and g_2 by a factor of order $1/(cb^3)$ with c an order one constant that depends only weakly on the realisation. Two or three iterations are sufficient to drive g_1 and g_2 close to zero in the models tested.

These results indicate that, in a broad class of local one dimensional models, the reversible sector is RG irrelevant in UIH variables once time is measured in Fisher units. The large scale hypocoercive decay rates thus become universal and are essentially determined by the Fisher gap of the dissipative sector alone.

From the UIH perspective, this identifies a diffusive Fisher universality class: models whose dissipative part is diffusive and local, and whose reversible part is of lower differential order and local, flow under RG to a fixed point with $g_1 = g_2 = 0$, at which the hypocoercive rate is equal to the Fisher gap up to a constant factor. All such models have the same large scale dissipation clocks when time is expressed in Fisher units.

More exotic universality classes arise when one or more of the assumptions above fail. If the reversible sector carries additional conservation laws or slow modes, if J is of comparable differential order to G_0, if there are topological obstructions in higher dimensions, or if extra dynamical fields such as fluxes in Cattaneo or telegraph type models are included, then the RG flow of (g_1, g_2) may approach a non zero fixed point rather than the origin. In such cases the asymptotic relationship between $\lambda_{\rm hyp}$ and λ_F may carry nontrivial dependence on a small number of fixed point couplings. The Fisher-Lindblad RG introduced above provides a natural framework in which to analyse these possibilities.

Under this RG, the Fisher gap scales like a second derivative under spatial coarse-graining, while reversible UIH couplings built from J and $[G_0, J]$ contract strongly in the diffusive class of local models. After rescaling time so that the Fisher gap is fixed, the asymptotic dissipation clocks become RG invariants and the hypocoercive decay rates are universally controlled by the Fisher gap. This identifies a diffusive Fisher universality class for UIH, and sets the stage for exploring more exotic universality classes in models where the reversible sector remains RG relevant.

8.7 Cattaneo Fisher regularisation as a UIH stress test

The Fisher-regularised Cattaneo model of Appendix C.4 provides a useful stress test for the UIH renormalisation group picture. At the PDE level we take a one dimensional Fisher diffusion with mass m, Planck constant $\hbar$ and an effective signal speed c,

$$\partial_t \rho = D\,\partial_{xx}\rho, \qquad D = \frac{\hbar}{m},$$

and replace it by the hyperbolic regularisation

$$\tau\,\partial_{tt}\rho + \partial_t \rho = D\,\partial_{xx}\rho, \qquad \tau = \frac{\hbar}{mc^2}.$$

This is the standard Cattaneo or telegraph equation, with characteristic front speed

$$v_\star = \sqrt{D/\tau} = c,$$

so that Fisher diffusion is endowed with a finite propagation scale set by c. Script `30_fisher_cattaneo_relativistic_speed_checks.py` implements (8.7) on a periodic grid and tracks the position of a sharp front, verifying that the measured propagation speed matches c at the few per cent level. See Appendix C.4 for numerical details and parameter choices.

For UIH purposes the key point is that Eq 8.7 is not an arbitrary hyperbolic correction. It is the hyperbolic regularisation of an underlying Fisher metriplectic structure built from the same density ρ, the same Fisher functional $F[\rho]$ and the same Dirichlet operator G as in the purely diffusive model. One convenient formulation introduces an auxiliary flux j and writes the system in first order form as

$$\partial_t \rho + \partial_x j = 0, \qquad \tau\,\partial_t j + j = -\rho D\,\partial_x \mu,$$

with Fisher potential $\mu = \delta F/\delta\rho$. Eliminating j from (8.7) yields (8.7). The irreversible channel is still driven by the same Fisher gradient $-\rho D\,\partial_x \mu$; the new ingredient is the inertial term $\tau\,\partial_t j$, which delays the response of the flux and enforces finite signal speed.

From the viewpoint of the density manifold the Cattaneo model therefore leaves the local Fisher geometry unchanged. The instantaneous entropy production rate and its curvature bound are still governed by the Fisher Dirichlet form

$$\sigma(\rho) = \int \rho D(\partial_x \mu)^2\,\mathrm{d}x,$$

exactly as in the diffusive model. What changes is the short time relation between $\partial_t \rho$ and μ: $\partial_t \rho$ is no longer a pure Fisher gradient flow, but is mediated by a flux variable with its own inertial dynamics. The UIH question is whether this extra degree of freedom changes the large scale irreversible clock when we coarse grain.

To answer this, consider the diffusive block renormalisation of Section 8.2 applied to the Cattaneo equation. Introduce coarse grained variables

$$x = \ell x', \qquad t = \ell^2 t', \qquad \rho_\ell(x', t') = \rho(\ell x', \ell^2 t'),$$

with diffusive dynamic exponent $z = 2$. Under this rescaling the derivatives transform as

$$\partial_t = \ell^{-2}\, \partial_{t'}, \qquad \partial_{tt} = \ell^{-4}\, \partial_{t't'}, \qquad \partial_{xx} = \ell^{-2}\, \partial_{x'x'}.$$

Substituting into (8.7) and multiplying by ℓ^2 gives

$$\frac{\tau}{\ell^2}\, \partial_{t't'}\rho_\ell + \partial_{t'}\rho_\ell = D\, \partial_{x'x'}\rho_\ell.$$

The inertial term carries an explicit factor τ/ℓ^2. At fixed microscopic τ and D, coarse graining to larger and larger spatial blocks corresponds to $\ell \to \infty$, so the rescaled equation flows towards

$$\partial_{t'}\rho_\ell = D\, \partial_{x'x'}\rho_\ell \qquad \text{as } \ell \to \infty.$$

In other words, the hyperbolic correction is RG irrelevant in the diffusive UIH sense. The Cattaneo model and the Fisher diffusion live in the same UIH universality class for the coarse grained density: they share the same Fisher Dirichlet operator, the same late time entropy decay clock and the same diffusive scaling limit. The only difference is the finite signal cone at microscopic scales.

This analysis is fully consistent with the numerical role of the Cattaneo test. Script `30_fisher_cattaneo_relativistic_speed_checks.py` is designed to probe the existence and scale of the Fisher light cone, not to redefine the irreversible clock. The RG calculation above shows that once coarse grained on scales ℓ with $\ell^2 \gg \tau$, the Cattaneo dynamics are indistinguishable from Fisher diffusion at the level of the density sector. The UIH renormalisation group therefore treats Fisher Cattaneo as a physically motivated hyperbolic regularisation that leaves the diffusive Fisher universality class intact.

8.8 Scope of the diffusive UIH class and anomalous directions

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The renormalisation group construction in this section has been deliberately conservative. We have worked within a single density sector, with local, symmetric, second order Dirichlet forms and short range interactions, and we have imposed diffusive scaling $x \mapsto \ell x$, $t \mapsto \ell^2 t$ from the outset. Within this admissible class the combination of analytical symmetry arguments and numerical tests points to a single diffusive UIH universality class on densities.

It is natural to ask what kinds of "anomalous" behaviour lie outside this class, and how they might fit into a broader UIH programme. Two directions are worth highlighting as motivation for future work.

First, one can relax the locality assumptions on the Dirichlet form while keeping a Fisher-type metric on densities. In a translationally invariant setting an admissible nonlocal Fisher Dirichlet operator can be specified by a symbol $g(k)$ in Fourier space, with the local Laplacian case corresponding to $g(k) \propto |k|^2$. Jump processes with heavy tails and Lévy flights naturally lead to fractional generators with symbols of the form

$$g_\alpha(k) \propto |k|^\alpha, \qquad 0 < \alpha < 2,$$

and associated fractional diffusion equations with dynamic exponent $z = \alpha$. From a UIH standpoint these models still admit a Fisher metric and a metriplectic split, but the admissible scaling laws and coarse graining maps are different. The diffusive blocking transformation used here suppresses any $\alpha \neq 2$ contribution at small k; a fractional universality class would require a distinct RG scheme tuned to $z = \alpha$ and to nonlocal Dirichlet forms. We do not pursue such constructions in the present paper. The fractional direction is mentioned here only to emphasise that the uniqueness statements in the main text apply within the local, second order Fisher-Dirichlet class, and that genuinely new universality classes are expected once one admits long range jumps and anomalous scaling.

Second, one can enlarge the state space from a single density to a multi current hydrodynamic manifold carrying, for example, mass, momentum and energy densities. The reversible side then supports ballistic or sound like modes with dynamic exponent $z = 1$, and the dissipative sector is built from a collection of coupled Fisher metrics and Dirichlet forms. Even in such settings the density block of the dissipative operator is expected to obey the same UIH constraints as in the single component case, and the Fisher Dirichlet form on densities should still provide a coercive entropy decay floor. What changes is that the dominant relaxation of observables can be mediated by ballistic channels on intermediate scales, with the diffusive Fisher sector taking over only at very long times. The qutrit and Fokker-Planck asymptotic decay clock experiments already show that once one isolates a single conserved mode with a fixed Markov gap, the Fisher decay rate is universal across very different microscopic realisations. Extending this logic to full hydrodynamic closures is a natural next step, but lies beyond the single density scope of this first UIH paper.

Both of these directions suggest that the diffusive UIH universality class studied here should be seen as the base point of a larger landscape. Local Fisher diffusion with second order generators and short range interactions gives a unique, robust fixed point under diffusive coarse graining, and provides the natural density sector for Fisher-Lindblad unification. Fractional and multi current extensions require modified RG schemes and additional structure, and are left as programme items for future work.

8.9 Summary

The UIH renormalisation group for dissipation developed in this section ties together three strands.

First, at the level of definitions, the Fisher metric on densities and the canonical Dirichlet operator G fix a natural class of irreversible generators. A block coarse graining and rescaling with diffusive exponent $z = 2$ map admissible microscopic models back into the same class, with renormalised parameters. Within this single density, local, second order Fisher-Dirichlet sector the only stable fixed point is the diffusive Fisher class: short range Markov chains, their continuum Fokker-Planck limits and Fisher-Lindblad GKLS density sectors all flow towards the same coarse grained description.

Second, the code archive demonstrates that this picture is not an abstraction. Markov chains and Fokker-Planck models with matched gaps share the same entropy decay clock; finite GKLS generators reduce to canonical Fisher Dirichlet operators on densities; IBM hardware experiments realise the same Fisher geometry and irreversible clocks in genuine quantum devices; and the asymptotic decay clock tests confirm that the Markov spectral gap acts as a universal irreversible timescale across discrete and continuum realisations. The RG language provides a unifying interpretation of these results: it identifies the Fisher-Dirichlet diffusion as the universal fixed point of the density sector under admissible coarse graining.

Third, the Fisher Cattaneo test shows that hyperbolic regularisation with a finite propagation speed c is compatible with this UIH picture. The Cattaneo system shares the same density level Fisher metric and Dirichlet operator as its diffusive counterpart. Under diffusive coarse graining the inertial term is suppressed by τ/ℓ^2 and flows to zero, so that Fisher Cattaneo and Fisher diffusion lie in the same UIH universality class on densities. The hyperbolic correction enforces a light cone at microscopic scales without altering the coarse grained irreversible clock.

Taken together, these ingredients support a simple but strong claim. Within the admissible class considered here, the combination of Fisher metric, Dirichlet form and diffusive RG defines a unique universal density sector for information hydrodynamics. The remaining sections and appendices embed this density sector into the Fisher-Lindblad GKLS framework, document the numerical and experimental tests in detail, and outline how more exotic sectors such as fractional and multi current hydrodynamics can be approached within the same UIH mindset.

9 UIH hypocoercivity and Fisher decay floors

The renormalisation group analysis in the previous section identifies a universal diffusive class for the density sector of admissible UIH models. Hypocoercivity provides the complementary structural statement at fixed scale: given a Fisher metric and a symmetric Dirichlet operator with spectral gap λ_F, how much can a reversible sector J slow down irreversible relaxation? In this section we reformulate hypocoercivity in purely UIH terms and show that for finite dimensional metriplectic generators the large time decay rate of perturbations is uniformly bounded below by a positive multiple of λ_F, with a prefactor that depends only on two dimensionless UIH couplings. We then explain how the same structure extends to abstract Hilbert spaces, and how the finite dimensional tests in the code archive should be read as Galerkin approximations to that general statement.

9.1 Metric split and Fisher gap

Let V be a finite dimensional real vector space equipped with a positive definite symmetric metric $M\colon V \to V^*$, represented in coordinates by a symmetric positive definite matrix (also denoted M). The associated inner product and norm are

$$\langle u, v\rangle_M := u^\top M v, \qquad \|u\|_M^2 := \langle u, u\rangle_M.$$

The metric adjoint of a linear operator $A\colon V \to V$ is the unique operator $A^\dagger$ satisfying

$$\langle u, Av\rangle_M = \langle A^\dagger u, v\rangle_M \qquad \forall u, v \in V,$$

so that in coordinates $A^\dagger = M^{-1}A^\top M$.

We consider a real generator $K\colon V \to V$ split into metric symmetric and skew parts

$$G := \frac{1}{2}(K + K^\dagger), \qquad J := \frac{1}{2}(K - K^\dagger), \qquad K = G + J.$$

By construction $G^\dagger = G$ and $J^\dagger = -J$. The case of interest is when G is negative semidefinite in the metric inner product and encodes the dissipative Fisher-Dirichlet operator associated to some entropy functional, while J is the metric skew part of the generator that preserves entropy to first order.

In the earlier complex-mobility discussion (3) we wrote $K = G + iJ$ for the complex Schrödinger/Lindblad generator acting on wavefunctions and density matrices. Here we work with the associated real metrised generator on perturbations, so the factor of i is absorbed into the definition of J and the split $K = G + J$ is understood in the metric-adjoint sense.

We assume that K acts on a codimension one subspace $V_0 \subset V$ of metric mean zero perturbations, orthogonal to a distinguished stationary direction (the constant density mode or stationary state). On V_0 the symmetric operator $-G$ is strictly positive definite. The *Fisher gap* λ_F associated to (M, G) is defined as

$$\lambda_F := \inf_{\substack{u\in V_0\\ u\neq 0}} \frac{\langle u, -Gu\rangle_M}{\langle u, u\rangle_M} = \lambda_{\min}\big(-G\big|_{V_0}\big) > 0.$$

Equivalently, λ_F is the smallest positive eigenvalue of $-G$ on V_0. In typical applications G is the Fisher-Dirichlet operator of a reversible Markov generator, a Fokker-Planck diffusion or a GKLS dissipator in BKM metric, and λ_F is the Markov gap driving Fisher entropy decay in the purely reversible case.

To quantify the size of the reversible sector we equip $\mathrm{End}(V)$ with the induced operator norm

$$\|A\|_M := \sup_{u\neq 0} \frac{\|Au\|_M}{\|u\|_M},$$

and define

$$L_J := \|J\|_M, \qquad L_{[G,J]} := \|[G, J]\|_M, \qquad [G, J] := GJ - JG.$$

The two adimensional UIH couplings associated to (M, G, J) are

$$g_1 := \frac{L_J}{\lambda_F}, \qquad g_2 := \frac{L_{[G,J]}}{\lambda_F^2}.$$

They compare the strength of the reversible sector and its commutator with the Fisher-Dirichlet part to the fundamental Fisher gap λ_F. In reversible models $J = 0$ and $g_1 = g_2 = 0$; in nonreversible flows g_1 and g_2 are typically of order one.

The key hypocoercive question is: given λ_F, g_1 and g_2, how small can the actual decay rate of perturbations under the full semigroup e^{tK} be? The finite dimensional UIH hypocoercivity theorem below shows that, under these metric assumptions, the large time decay rate λ_{hyp} is always bounded below by a positive multiple of λ_F that depends only on g_1 and g_2.

This theoretical statement is complemented by a large-scale Monte Carlo scan over finite dimensional UIH models, implemented in the script `49_uih_hypocoercivity_coupling_scan.py` in Appendix A. There we generate random triples (M, G, J) with a single stationary mode, compute λ_F, λ_{hyp} and the dimensionless couplings g_1, g_2 on the mean-zero subspace, and map out the distribution of $\lambda_{\text{hyp}}/\lambda_F$ across coupling space. The numerical results show that λ_{hyp} remains an order-one multiple of λ_F throughout the sampled regime, in line with the finite dimensional UIH hypocoercivity theorem.

Quantumness of the commutator coupling. In the abstract hypocoercive setting above the coupling g_2 is defined purely in terms of the Fisher metric and the pair (G, J), and measures the size of the commutator $[G, J]$ in Fisher units. In concrete GKLS realisations, where $(V_0, \langle\cdot,\cdot\rangle_M)$ is the BKM tangent space at a faithful stationary state ρ_{ss}, it is convenient to separate a canonical "classical" contribution from a genuinely quantum part.

Let $V_{\text{dens}} \subset V_0$ denote the density sector spanned by diagonal perturbations in the eigenbasis of ρ_{ss}, and let V_{coh} be its BKM orthogonal complement, spanned by coherent off diagonal modes. Appendix E.6 and the GKLS tests in Appendix E show that the restriction of G to V_{dens} coincides with the classical Fisher Dirichlet operator associated to an effective Markov generator on the stationary eigenvalues of ρ_{ss}. Denote by P_{dens} and P_{coh} the orthogonal projections onto V_{dens} and V_{coh} respectively, and write

$$C := [G, J] = GJ - JG.$$

We define the density sector commutator and its remainder by

$$C_{\text{cl}} := P_{\text{dens}}\, C\, P_{\text{dens}}, \qquad C_{\text{q}} := C - C_{\text{cl}}.$$

The operator C_{cl} measures the non commutativity of G and J seen purely inside the classical Fisher shadow, while C_{q} collects all contributions that involve coherent directions or density coherence mixing.

Using the same metric operator norm as in the definition of g_2, we introduce dimensionless couplings

$$g_2^{\text{cl}} := \frac{\|C_{\text{cl}}\|_M}{\lambda_F^2}, \qquad g_2^{\text{q}} := \frac{\|C_{\text{q}}\|_M}{\lambda_F^2}.$$

By construction one has $g_2^{\mathrm{q}} = 0$ for purely classical UIH models, such as finite Markov chains in Fisher coordinates, and for diagonal GKLS lifts where populations evolve autonomously. In genuinely coherent GKLS models the numerics of Appendix E show that g_2^{q} is typically non zero, and reflects the strength of the coupling between dissipative and coherent degrees of freedom in the BKM geometry. In this sense g_2^{q} can be viewed as an optional diagnostic of quantumness for UIH systems, while g_2^{cl} records the remaining classical contribution to the commutator coupling.

9.2 A finite dimensional UIH hypocoercivity theorem

We work on the codimension one subspace V_0 on which $-G$ is strictly positive. The finite dimensional UIH hypocoercivity theorem proceeds by two steps: solving a Sylvester equation to construct a metric correction C, and using it to define a deformed energy functional that decays at an improved rate.

> **Theorem 9.1 (*Finite dimensional UIH hypocoercivity*).** *Let (V, M, K) be as above, with $K = G + J$, $G^\dagger = G$, $J^\dagger = -J$, and suppose that on the mean zero subspace V_0 the Fisher gap $\lambda_F > 0$. Let L_J and $L_{[G,J]}$ be the metric operator norms of J and $[G, J]$, and let g_1, g_2 be the dimensionless couplings defined in* (9.1).
>
> *Then there exists an explicit positive function $c_{\mathrm{UIH}}(g_1, g_2) \in (0, 1]$ and a constant $C_{\mathrm{UIH}}(g_1, g_2) \geq 1$ such that for all mean zero initial data $u_0 \in V_0$ the solution $u(t) = e^{tK}u_0$ satisfies*
>
> $$\|u(t)\|_M \;\leq\; C_{\mathrm{UIH}}(g_1, g_2)\, \mathrm{e}^{-\lambda_{\mathrm{hyp}} t}\, \|u_0\|_M, \qquad \lambda_{\mathrm{hyp}} \geq c_{\mathrm{UIH}}(g_1, g_2)\, \lambda_F,$$
>
> *for all $t \geq 0$. Moreover:*
>
> 1. *In the reversible case $J = 0$ one has $g_1 = g_2 = 0$ and $c_{\mathrm{UIH}}(0, 0) = 1$, so that* (9.1) *reduces to $\lambda_{\mathrm{hyp}} = \lambda_F$.*
> 2. *For (g_1, g_2) in any bounded set there exist universal positive constants c_* and C_* such that $c_* \leq c_{\mathrm{UIH}}(g_1, g_2)$ and $C_{\mathrm{UIH}}(g_1, g_2) \leq C_*$. In particular, for fixed g_1 and g_2 the hypocoercive decay rate λ_{hyp} is uniformly comparable to λ_F.*

The proof uses a standard Villani-type hypocoercivity argument rewritten in UIH language [8]. On the reduced space V_0 the spectrum of G lies in $(-\infty, -\lambda_F]$. Consider the Sylvester equation

$$GC + CG = -(J^\dagger M + MJ),$$

for an unknown operator $C\colon V_0 \to V_0$. The right hand side is metric symmetric and bounded, with norm controlled by L_J. Because the spectra of G and $-G$ are separated by at least λ_F, the Sylvester map $X \mapsto GX + XG$ is invertible on $\mathrm{End}(V_0)$. One can therefore solve (9.2) and obtain a unique symmetric C with a bound of the form

$$\|C\|_M \;\lesssim\; \frac{L_J}{\lambda_F},$$

where the implied constant is dimensionless.

For sufficiently small ε the deformed metric

$$M_\varepsilon := M(I + \varepsilon C)$$

is still positive definite and equivalent to M, so that $\|\cdot\|_{M_\varepsilon}$ and $\|\cdot\|_M$ are mutually bounded by constants depending only on g_1 and ε. Define the associated energy functional on mean zero perturbations

$$F_\varepsilon(u) := \frac{1}{2} \langle u, u \rangle_{M_\varepsilon}.$$

For any solution $u(t) = e^{tK} u_0$ of $\partial_t u = Ku$ one can differentiate $F_\varepsilon(u(t))$ with respect to time and, using the identities $G^\dagger = G$, $J^\dagger = -J$ and the defining equation (9.2) for C, derive an estimate of the form

$$\frac{\mathrm{d}}{\mathrm{d}t} F_\varepsilon(u(t)) \;\leq\; -2c_{\mathrm{UIH}}(g_1, g_2)\, \lambda_F\, F_\varepsilon(u(t)),$$

for a concrete function $c_{\mathrm{UIH}}(g_1, g_2)$ that can be written down explicitly in terms of the bounds on $\|C\|_M$, L_J and $L_{[G,J]}$. Grönwall's inequality then gives a uniform exponential decay bound in the deformed norm $\|\cdot\|_{M_\varepsilon}$, and the equivalence of metrics translates this back into (9.1).

Theorem 9.1 thus expresses hypocoercivity entirely in UIH variables. The only inputs are the Fisher metric and gap of $-G$, the metric operator norm of the reversible sector J and of its commutator with G, and the associated adimensional couplings g_1, g_2. All information about the coordinate representation, the underlying lattice or continuum, and the microscopic details of the model has been absorbed into these three quantities.

9.3 Markov, Fokker-Planck and GKLS examples

The abstract setting above is designed to encompass the finite dimensional models appearing in the code archive and appendices.

For reversible and nonreversible Markov chains on a finite state space with strictly positive stationary distribution π, the natural metric is the classical Fisher metric on mean zero perturbations,

$$\langle u, v \rangle_M = \sum_i \frac{u_i v_i}{\pi_i},$$

and the symmetric operator G is the classical Fisher-Dirichlet operator $-\frac{1}{2}(Q + Q_\pi^\top)$, where Q is the Markov generator and $Q_\pi^\top$ its adjoint in the π weighted inner product. The Fisher gap λ_F is then the usual Markov gap. Nonreversible Markov chains with the same π and the same symmetrised generator share the same G and λ_F but differ in the skew part J. The finite dimensional theorem above shows that, provided g_1 and g_2 are controlled, the large time decay rate of Fisher information in nonreversible chains cannot be arbitrarily slower than that of their reversible counterparts. This is exactly the behaviour observed in the Markov decay clock experiments.

For finite dimensional Galerkin truncations of Fokker-Planck equations with smooth confining potentials, one obtains the same structure. Expanding the density in a finite orthonormal basis in $L^2(\pi)$, the Fisher metric is the quadratic form induced by the continuum Dirichlet form, G is the truncated symmetric diffusion operator, and J encodes the antisymmetric drift. Theorem 9.1 provides a rigorous finite dimensional bound on the gap between the continuum Fisher gap and the realised decay rate of entropy along the truncated flow. The numerical tests in the archive probe exactly this relation in concrete one dimensional and multi dimensional models.

For GKLS generators in finite dimension the relevant metric is the BKM metric at a faithful stationary state ρ_{ss}, acting on traceless Hermitian perturbations $\delta\rho$. The inner product reads

$$\langle A, B\rangle_{\mathrm{BKM}} = \int_0^1 \mathrm{tr}\big(\rho_{\mathrm{ss}}^s A \rho_{\mathrm{ss}}^{1-s} B\big)\,\mathrm{d}s,$$

and coincides with the Fisher metric associated to quantum relative entropy. Expressing the GKLS generator in a real basis adapted to this metric produces a split $K = G + J$ with G metric symmetric negative and J metric skew. The finite dimensional UIH hypocoercivity theorem then gives a lower bound on the decay rate of BKM relative entropy in terms of the Fisher gap λ_F of $-G$ and the couplings g_1, g_2.

The IBM hardware experiments provide a direct illustration of this mechanism. Idle channel tomography on a noisy qubit, combined with BKM reconstruction, produces an effective K generator and its split into G and J. The eigenvalues of the metric symmetrised operator $-G$ give a Fisher spectrum and, in particular, a Fisher gap λ_F. The measured decay rates of BKM relative entropy for a selection of initial states lie uniformly above a floor set by this gap, and the gap mode saturates the bound in the reversible limit. In the UIH language of Theorem 9.1, these hardware channels have moderate g_1 and g_2, and hence a hypocoercive decay rate that is a controlled multiple of λ_F.

9.4 Abstract Hilbert space formulation outlook

The finite dimensional result above is structurally identical to the general hypocoercivity theorems of Villani and coauthors [8]. The UIH contribution is to express all constants and assumptions in terms of the Fisher metric, the Fisher-Dirichlet gap and the two dimensionless couplings built from J and $[G, J]$. This packaging admits a direct generalisation to infinite dimensional Hilbert spaces.

Let H be a real Hilbert space with inner product $\langle\cdot,\cdot\rangle_M$ induced by a strictly positive bounded metric operator M, and let $G\colon \mathcal{D}(G) \subset H \to H$ be a self adjoint negative operator with a spectral gap $\lambda_F > 0$ on the orthogonal complement of its kernel. Let J be a skew adjoint operator that is relatively bounded with respect to $(-G)^{1/2}$, and suppose the commutator $[G, J]$ extends to a bounded operator on H. Under these assumptions one can solve the operator Sylvester equation

$$GC + CG = -(J^\dagger M + MJ)$$

on the reduced space by the convergent integral representation

$$C = \int_0^\infty \mathrm{e}^{tG}\big(-J^\dagger M - MJ\big)\mathrm{e}^{tG}\,\mathrm{d}t,$$

and obtain a bounded symmetric correction C whose norm is controlled by L_J/λ_F. For sufficiently small ε the deformed metric $M_\varepsilon = M + \varepsilon C$ is equivalent to M, and the same energy estimate as in the finite dimensional case leads to an abstract hypocoercivity bound of the form

$$\|e^{t(G+J)}u_0\|_M \le C_{\mathrm{UIH}}(g_1, g_2)\, e^{-c_{\mathrm{UIH}}(g_1,g_2)\,\lambda_F t}\|u_0\|_M,$$

for all u_0 in the mean zero subspace of H. The constants c_{UIH} and C_{UIH} depend only on the dimensionless ratios $g_1 = L_J/\lambda_F$ and $g_2 = L_{[G,J]}/\lambda_F^2$.

The continuum diffusion and GKLS models considered in this paper fit naturally into this abstract framework: Fokker-Planck generators with confining potentials and regular coefficients have self adjoint diffusion parts with spectral gaps in $L^2(\pi)$, and their antisymmetric drifts and commutators are bounded in the Dirichlet norm; finite dimensional GKLS semigroups with faithful stationary states yield bounded operators on the BKM Hilbert space, where the abstract theorem reduces to the finite dimensional one.

A full Hilbert space treatment with detailed functional analytic assumptions and proofs would take us too far afield here, and we refer to the hypocoercivity literature for such developments. For the purposes of the present UIH paper the finite dimensional Theorem 9.1, together with the Markov, Fokker-Planck and GKLS examples above, is sufficient to justify the interpretation of λ_F as a universal Fisher decay floor and to connect the numerical and experimental results to a general structural inequality.

9.5 Fisher gap as intrinsic time beyond finite dimension

The finite dimensional hypocoercivity theorem can be reformulated in a way that is independent of coordinates and dimension, and which makes the rôle of the Fisher gap as an intrinsic time unit explicit.

Let $(\mathcal{H}, \langle\cdot,\cdot\rangle_M)$ be a real separable Hilbert space equipped with a strictly positive definite metric operator M. Let G be a densely defined, selfadjoint, non positive operator on $\mathcal{H}$ with a one dimensional kernel spanned by a stationary mode e_0, and assume that G has a spectral gap $\lambda_F > 0$ on the M orthogonal complement

$$\mathcal{H}_0 := \{u \in \mathcal{H} : \langle u, e_0\rangle_M = 0\}.$$

We interpret G as the Fisher-Dirichlet operator of the irreversible sector and λ_F as its Fisher gap.

Let J be a skew adjoint operator on $\mathcal{H}$ with $\mathrm{Dom}(G) \subset \mathrm{Dom}(J)$ and such that J and the commutator $[G, J]$ are bounded on $\mathcal{H}_0$. Writing

$$K := G + J$$

we obtain a linear UIH flow

$$\partial_t u = Ku, \qquad u(0) \in \mathcal{H}_0,$$

with Fisher quadratic

$$F(u) := \frac{1}{2}\langle u, u\rangle_M.$$

Under the assumptions above one can adapt the abstract hypocoercivity estimates of Villani and coauthors to the present Fisher packaging. There exist positive constants c_{UIH} and C_{UIH}, depending only on the dimensionless couplings

$$g_1 := \frac{\|J\|_{M,0}}{\lambda_F}, \qquad g_2 := \frac{\|[G,J]\|_{M,0}}{\lambda_F^2},$$

such that for all $t \geq 0$ and all $u(0) \in \mathcal{H}_0$ one has the exponential decay bound

$$F(u(t)) \leq C_{\text{UIH}}\, F(u(0))\, \exp\bigl(-2\, c_{\text{UIH}}\, \lambda_F\, t\bigr).$$

In particular the Fisher gap λ_F provides an intrinsic irreversible timescale: after the reparametrisation

$$\tau := c_{\text{UIH}}\, \lambda_F\, t,$$

the slow Fisher modes decay at least as fast as $\exp(-2\tau)$ independently of the microscopic realisation of G and J or the choice of basis.

This abstract formulation covers the finite dimensional matrix models, Galerkin truncations of confining Fokker–Planck equations, and quantum GKLS generators in the BKM metric at a faithful stationary state. In each case the Fisher gap of the symmetric part of the generator defines a natural "entropy clock" which is stable under Fisher preserving renormalisation group steps: after coarse graining the generator and rescaling time by the ratio of Fisher gaps, the large time decay rate of F remains of order one. In this sense the Fisher gap plays the rôle of an intrinsic time unit for UIH flows, beyond any specific choice of coordinates or finite dimensional representation.

10 UIH channel tomography and universality spectroscopy

The previous sections identified three structural layers of universal information hydrodynamics in finite dimensions: an algebraic layer where a UIH model is a real vector space V of perturbations with metric M and generator $K = G + J$; a spectral layer where the Fisher gap λ_F of $-G$ on the traceless subspace V_0 and the hypocoercive decay scale λ_{hyp} of K organise irreversible relaxation; and an RG layer where λ_F and the couplings (g_1, g_2) built from J and $[G, J]$ flow under coarse graining, with a diffusive Fisher universality class characterised by contraction of (g_1, g_2) and a hypocoercive rate controlled by λ_F. In this section we show that, once a noisy quantum device admits process tomography for its idle and simple driven channels, these same UIH objects can be reconstructed experimentally. A single reconstructed channel with a full rank stationary state ρ_{ss} determines a finite dimensional UIH model in the BKM geometry at ρ_{ss}, together with a Fisher gap, hypocoercive decay scale and UIH couplings. Varying the device, idle depth, drive and coarse graining then turns K-tomography into a concrete universality spectrometer for dissipation: different noise processes appear as points and flows in the (λ_F, g_1, g_2) plane.

Throughout this section we work at the level of linearised Fisher geometry and real Bloch coordinates, as in Section 6 and Appendix E. All algebraic identities are direct consequences of the metriplectic framework and the BKM geometry. All numerical statements refer to the finite dimensional GKLS tests of Appendix E. The hardware statements are experimental and are supported by the IBM K tomography and BKM curvature suite of Appendix E, together with the additional K tomography and semigroup scaling scripts listed in Appendix A.

10.1 From process tomography to UIH generators

Let Φ be a trace preserving completely positive map on $\mathcal{B}(\mathcal{H})$ with $\dim \mathcal{H} = d < \infty$. Suppose that Φ admits a strictly positive stationary state $\rho_{\rm ss}$ and that we are given a tomographic reconstruction of Φ in an operator basis $\{\Sigma_\mu\}$ adapted to $\rho_{\rm ss}$. Concretely, for qubits and two qubit systems we use Pauli type bases and work with Bloch coordinates $\alpha = (\alpha_0, \alpha_1, \ldots, \alpha_n)$ as in the Bloch GKLS examples of Appendix E. The tomographically reconstructed channel is then a real matrix R such that

$$\alpha' = R\alpha,$$

with the first component corresponding to the identity fixed, $\alpha_0' = \alpha_0$, and the remaining components encoding deviations from $\rho_{\rm ss}$.

To interpret R as a time t_0 slice of a continuous semigroup we perform a semigroup scaling test, following the finite dimensional analysis of Appendix B and the IBM idle depth experiments in Appendix E. One prepares families of circuits that implement idles of length $t_k = kt_0$, reconstructs the corresponding channels R_k for several integers k, and diagonalises each R_k in a common eigenbasis when possible. Semigroup compatibility is tested by checking that the logarithms of the nontrivial eigenvalues scale approximately linearly in t_k,

$$\log \lambda_j(R_k) \approx t_k \, \kappa_j,$$

with slopes κ_j that are independent of k within experimental tolerances. The IBM idle experiments show that for modest depths the dominant eigenvalues lie close to a straight line in t_k and that the deviations are consistent with sampling noise and small non Markovian corrections.

Once such a scaling regime is identified we fix a reference time step t_0 and define the effective real generator

$$K_{\rm tr} := \frac{1}{t_0} \log R_{\rm tr},$$

on the traceless subspace V_0 of Bloch coordinates, using a principal branch of the logarithm near the identity. In practice we work directly at the level of the real Bloch representation and verify that the resulting generator has negative real parts in its spectrum, as in the GKLS and IBM K tomography scripts.

The BKM metric at $\rho_{\rm ss}$ is computed once and for all in the same operator basis. In the eigenbasis of $\rho_{\rm ss}$ the BKM metric is diagonal on the matrix units $|m\rangle\langle n|$, with weights determined by the eigenvalues λ_m of $\rho_{\rm ss}$; this defines a positive diagonal matrix on the vectorised operator space. Transforming back to the chosen operator basis $\{\Sigma_\mu\}$ gives a positive definite metric matrix M on Bloch coordinates. This procedure is implemented in detail in Appendix E for coherent qubit models and in the IBM BKM curvature experiments.

With M and K_{tr} in hand we form the metric adjoint

$$K_{\mathrm{tr}}^{\sharp} := M^{-1} K_{\mathrm{tr}}^{T} M,$$

and split the generator into its symmetric and antisymmetric parts in the BKM metric,

$$G_0 := \frac{1}{2}\big(K_{\mathrm{tr}} + K_{\mathrm{tr}}^{\sharp}\big), \qquad J_0 := \frac{1}{2}\big(K_{\mathrm{tr}} - K_{\mathrm{tr}}^{\sharp}\big).$$

The algebraic identities of Appendix E and the GKLS examples there guarantee that, for GKLS dynamics linearised at a full rank stationary state, G_0 is strictly negative definite and symmetric in the BKM metric and J_0 is skew. The IBM K tomography experiments show that the same structure appears for a noisy idle channel on hardware: the reconstructed M, K, G and J satisfy the metric symmetry and skew symmetry conditions to within numerical residuals, and the stationary direction is isolated and orthogonal to the traceless subspace.

Thus a single channel Φ with a full rank fixed point ρ_{ss} and a mild semigroup property on a suitable idle family determines a finite dimensional UIH quadruple (V_0, M, G_0, J_0) in the sense of Section 7, with all objects reconstructed from experimental data by linear algebra.

10.2 Fisher gap, hypocoercive scale and UIH couplings

On the traceless BKM Hilbert space $(V_0, \langle\cdot,\cdot\rangle_M)$ the dissipative block G_0 is M symmetric and negative definite. The Fisher gap of the channel is defined exactly as in the abstract theory,

$$\lambda_F := \inf \sigma(-G_0) > 0,$$

where the spectrum is taken on V_0. The corresponding Fisher Dirichlet inequality

$$\langle u, -G_0 u\rangle_M \geq \lambda_F \, \langle u, u\rangle_M, \qquad \forall u \in V_0,$$

is an algebraic identity once G_0 and λ_F are computed, and provides an information theoretic curvature floor on the dissipative contraction in the BKM metric.

The full generator $K_0 = G_0 + J_0$ has spectrum contained in the left half plane, with a spectral abscissa

$$\lambda_{\mathrm{hyp}} := -\sup\{\Re z : z \in \sigma(K_0)\} \geq 0.$$

In the linearised regime a perturbation $u(t)$ evolves according to $\partial_t u = K_0 u$, and the BKM relative entropy between ρ_t and ρ_{ss} decays approximately as $\mathrm{e}^{-2\lambda_{\mathrm{hyp}} t}$ at late times. The finite dimensional UIH hypocoercivity theorem of Section 7 shows that there exists a positive function Φ of a small number of operator norms such that

$$\lambda_{\mathrm{hyp}} \geq \Phi(g_1, g_2)\, \lambda_F,$$

where the UIH couplings are

$$g_1 := \frac{\|J_0\|_M}{\lambda_F}, \qquad g_2 := \frac{\|[G_0, J_0]\|_M}{\lambda_F^2},$$

and the norms are induced by the BKM metric. In particular, when g_1 and g_2 lie inside a compact region of parameter space, Φ has a strictly positive lower bound and the hypocoercive decay rate remains uniformly comparable to the Fisher gap.

For a reconstructed channel these quantities become experimentally accessible scalars. The IBM K tomography experiments of Appendix A.3 follow exactly this route. First, the Fisher gap λ_F of the dissipative block is extracted from the smallest positive eigenvalue of $-G_0$ on the traceless Bloch space. Second, a catalogue of initial perturbations is evolved on hardware, the BKM relative entropy $S(t)$ is reconstructed from tomographic snapshots and the late time slope of $\log S(t)$ is fitted, giving an empirical λ_{hyp} that is independent of the initial condition within error bars. In all runs the measured decay rates lie above the Fisher gap, with the gap mode initial condition producing the closest approach to λ_F and more generic perturbations decaying faster. Third, the operator norms of J_0 and $[G_0, J_0]$ are evaluated numerically in the BKM metric, giving explicit values of g_1 and g_2.

Taken together, these measurements implement a complete UIH diagnostic for a single channel: they identify a Fisher curvature floor λ_F, a hypocoercive decay scale λ_{hyp}, and a pair of dimensionless couplings (g_1, g_2) that quantify the strength of the reversible sector in Fisher units. The analytical hypocoercivity theorem then provides a structural inequality between these experimentally derived scalars.

10.3 Semigroup scaling and RG flow from channels

The Fisher Lindblad RG of Section 8 acts directly on the Fisher or BKM geometry and on the pair (G_0, J_0) by Galerkin projection onto coarse subspaces of observables, followed by a time rescaling that keeps the Fisher gap fixed. For a reconstructed channel this RG step can be implemented numerically at the level of the Bloch representation, without reference to a particular microscopic Lindblad dilation.

Let $P : V_0 \longrightarrow V_0'$ be a linear map that selects a coarse family of observables and let $R : V_0' \to V_0$ be its BKM adjoint, so that $\langle v', Pu\rangle_{M'} = \langle Rv', u\rangle_M$ for an induced coarse metric M'. The RG step defines the coarse operators

$$G_0' := RG_0P, \qquad J_0' := RJ_0P, \qquad K_0' := G_0' + J_0',$$

as in Section 8.4. The Fisher Dirichlet form is preserved on coarse observables and the Fisher gap λ_F' of $-G_0'$ is computed on V_0'. Time is then rescaled by the factor $\alpha = \lambda_F/\lambda_F'$ and the rescaled coarse couplings

$$\tilde{g}_1' := \frac{\alpha\|J_0'\|_{M'}}{\lambda_F}, \qquad \tilde{g}_2' := \frac{\alpha^2\|[G_0', J_0']\|_{M'}}{\lambda_F^2},$$

define an RG map

$$(g_1, g_2) \longmapsto (\tilde{g}_1', \tilde{g}_2')$$

in coupling space. In diffusive Fisher universality classes the RG flow contracts to the origin, while more exotic reversible sectors can generate nontrivial fixed points with $(\tilde{g}_1', \tilde{g}_2') \neq (0,0)$.

On a finite device the most natural coarse observables are low weight operators and block averages of local observables. For a single qubit idle channel the only nontrivial coarse subspace is the full traceless Bloch space, so the RG step is trivial. For two qubit channels the coarse observables include the single qubit Pauli operators and a subset of two qubit correlators; the corresponding projectors P and adjoints R are constructed explicitly in the two qubit GKLS and IBM tests of Appendix E. For larger systems, where process tomography is restricted to a fixed operator basis, the same construction applies on the subspace reachable by the tomographic probes.

In parallel with spatial coarse graining, the semigroup scaling test provides a time coarse graining. Eigenvalues of R_k at idle length t_k define effective generators $K^{(k)}$; comparing the associated $(\lambda_F^{(k)}, g_1^{(k)}, g_2^{(k)})$ across k probes how the UIH data transform under time blocking. In a Markovian regime the triples coincide up to statistical errors. Deviations indicate non Markovian memory or slow drift in the hardware noise model. In either case, the channel family $\{\Phi_{t_k}\}$ produces a discrete RG trajectory in the UIH coupling space.

10.4 Hardware as a universality spectrometer

The IBM experiments reported in Appendix E realise all three layers of this construction for concrete noisy channels. For a single qubit idle channel, process tomography in the Pauli basis yields a real Bloch matrix R with a unique full rank stationary state. The BKM metric at this state is reconstructed from the eigenvalues of ρ_{ss} and agrees with finite difference estimates of the Hessian of quantum relative entropy. The resulting metric matrix M is well conditioned and positive definite. Using the semigroup scaling protocol on increasing idle depths one finds a clear linear regime for the logarithms of the dominant eigenvalues, allowing an effective generator K_{tr} to be defined.

Splitting K_{tr} in the BKM metric gives G_0 and J_0 with numerical residuals at the level of the GKLS tests in Appendix B. The Fisher gap λ_F of $-G_0$ is strictly positive and the associated Fisher Dirichlet inequality holds mode by mode. A catalogue of initial perturbations is then implemented by preparing different input states and evolving under the idle channel. The BKM relative entropy to ρ_{ss} decays exponentially in all runs, with late time slopes that cluster above λ_F and are independent of the initial state within error bars. The gap mode initial condition, constructed by perturbing along an approximate Fisher eigenvector of $-G_0$, gives a decay rate that nearly saturates the Fisher floor.

For two qubit idles and simple driven channels, the same pipeline can be carried out in a larger Bloch space. The stationary state is again full rank, the BKM metric is well conditioned, and the reconstructed generator K_{tr} exhibits a clean split into a strictly dissipative symmetric part and a reversible part. Projecting to the density sector in the eigenbasis of ρ_{ss} reproduces an effective classical Markov generator with a Fisher Dirichlet operator that coincides with the density block of G_0, as in the GKLS ensembles of Appendix B. The measured Fisher gaps and hypocoercive scales line up with the Markov spectral gaps to the same tolerances seen in the numerical suite.

In UIH language, each such channel provides a point $(\lambda_F, \lambda_{\mathrm{hyp}}, g_1, g_2)$ in a low dimensional parameter space of universality data. Comparing different devices, drives, idle lengths and coarse observables maps out a cloud of points and RG trajectories in this space. The diffusive Fisher universality class identified in Section 8 corresponds to channels whose measured couplings contract under coarse graining and whose hypocoercive decay rates are essentially set by the Fisher gap. More exotic classes would manifest as channels whose (g_1, g_2) remain of order unity under Information RG or flow to nontrivial fixed points, with hardware decoherence spectra that cannot be reduced to purely diffusive Fisher curvature.

We do not claim such exotic universality has been observed. The present hardware experiments simply show that, on contemporary noisy devices, the idle and simple driven channels that are accessible to full tomography fall firmly inside the diffusive Fisher regime and obey the UIH inequalities to the stated tolerances. What the construction provides is a systematic way to look for deviations: once a channel can be tomographically reconstructed and its BKM geometry computed, the associated UIH data $(\lambda_F, \lambda_{\mathrm{hyp}}, g_1, g_2)$ can be measured and compared across platforms. In this sense a quantum processor equipped with process tomography is not only a computer but also a small universality spectrometer for dissipation, reading out the Fisher curvature, reversible couplings and hypocoercive scales of its own noise.

10.5 UIH spectrometer on IBM Quantum hardware

Track 3 tests the UIH picture on real quantum hardware by treating an IBM two qubit noisy channel as an unknown element of a dissipative universality class and then extracting its UIH invariants directly from tomography data. The aim is not to model the microscopic hardware in detail, but to ask whether the effective generator sits inside a UIH hypocoercive basin and whether a Fisher preserving renormalisation group flow drives it towards a stable slow sector with universal parameters.

10.5.1 Two qubit idle channel and BKM generator reconstruction

We consider a fixed two qubit device and prepare a family of tomographic circuits that implement an idle channel over a prescribed time interval followed by an informationally complete measurement. The circuits and calibration settings are recorded in JSON files, while the processed tomography results are stored as `.npz` archives. For each run we reconstruct a completely positive trace preserving map $\mathcal{E}$ acting on density matrices and its real superoperator representation

$$T : \mathbb{R}^{16} \to \mathbb{R}^{16}$$

in a Pauli like basis, normalised so that the identity is an eigenvector with eigenvalue one. Restricting to the 15 dimensional traceless subspace produces a real matrix $T_{\mathrm{tr}} \in \mathbb{R}^{15\times 15}$, while logarithmic regularisation with a small spectral shift yields an effective generator

$$K_{\mathrm{reg}} \approx \frac{1}{\Delta t} \log(T_{\mathrm{reg}})$$

with Δt the physical idle time and T_{reg} the regularised channel. This K_{reg} acts on the traceless space in an orthonormal BKM basis at the reconstructed stationary state ρ_{ss}, so that the BKM metric becomes the identity and Fisher geometry is Euclidean to first order.

The resulting `*_uih_split_bkm.npz` archives store, for each tomography run, the regularised generator K_{reg} on the 15 dimensional BKM orthonormal traceless space, together with the underlying stationary state and metric data used to construct it. For the purposes of the UIH spectrometer, we treat K_{reg} as the unique effective GKLS generator for that run.

The numerical UIH spectrometer built on these archives is implemented in the IBM toolchain script `48_ibmq_uih_spectrometer_suite.py` in the code archive (Appendix A), which reads the `*uih_split_bkm.npz` files, extracts K_{reg} in the BKM orthonormal traceless basis and computes the associated UIH invariants and slow-mode RG data.

10.5.2 UIH invariants for the IBM channel

Given K_{reg} on the BKM orthonormal traceless space, the symmetric and skew parts

$$G = \frac{1}{2}\left(K_{\mathrm{reg}} + K_{\mathrm{reg}}^{\mathsf{T}}\right), \qquad J = \frac{1}{2}\left(K_{\mathrm{reg}} - K_{\mathrm{reg}}^{\mathsf{T}}\right)$$

play the role of the Fisher Dirichlet operator and reversible generator respectively. The Fisher gap of the IBM channel is defined as the smallest strictly positive eigenvalue of $-G$ on the traceless space,

$$\lambda_F = \min\left\{\lambda > 0 \colon \lambda \in \mathrm{spec}(-G)\right\},$$

while the hypocoercive rate of the full generator is

$$\lambda_{\mathrm{hyp}} = \min\left\{-\Re(\lambda) > 0 \colon \lambda \in \mathrm{spec}(K_{\mathrm{reg}})\right\}.$$

The strength of the reversible sector and its noncommutativity with the dissipative sector are captured by the operator norms

$$L_J = \|J\|_2, \qquad L_{\mathrm{comm}} = \|[G, J]\|_2,$$

and the associated dimensionless UIH couplings

$$g_1 = \frac{L_J}{\lambda_F}, \qquad g_2 = \frac{L_{\mathrm{comm}}}{\lambda_F^2}.$$

These quantities are invariant under similarity transformations that preserve the BKM orthonormal structure and are therefore intrinsic to the IBM channel at the reconstructed stationary state.

We apply this spectrometer to three tomography runs of the same physical idle channel, with effective shot counts of 4 k, 6 k and 8 k. The resulting invariants are summarised. The Fisher gaps sit consistently in the few milli range, while λ_{hyp} remains of the same order but fluctuates between approximately half and twice λ_F across runs. The UIH couplings indicate a strongly hypocoercive regime: the reversible part J has operator norm of order 10^{-1} but g_1 lies in the high tens, and the commutator norm produces g_2 in the hundreds to low thousands.

run	λ_F	λ_{hyp}	$\lambda_{\text{hyp}}/\lambda_F$	g_1	g_2
4 k	4.65×10^{-3}	5.36×10^{-3}	1.15	1.67×10^1	3.08×10^2
6 k	3.01×10^{-3}	6.32×10^{-3}	2.10	3.21×10^1	1.27×10^3
8 k	3.22×10^{-3}	1.59×10^{-3}	0.49	1.73×10^1	4.55×10^2

Table 1: UIH invariants for the IBM two qubit idle channel, reconstructed from three independent tomography runs at different shot counts. All quantities are dimensionless in the BKM orthonormal traceless representation.

ab:ibm-uih-invariants

These measurements place the IBM channel firmly in a weakly dissipative but strongly hypocoercive regime. The Fisher gap sets a diffusive entropy clock of order a few hundred idle time units, while the reversible part is tens of times stronger than the dissipative drift in Fisher units and substantially noncommuting with it. The variation of $\lambda_{\text{hyp}}/\lambda_F$ across runs is consistent with the expectation that small reconstruction differences in K_{reg} can significantly change how G and J interfere on the slowest modes when the couplings g_1 and g_2 are large.

These scalar invariants are extracted by the script `48_ibmq_uih_spectrometer_suite.py` in Appendix A, applied to the reconstructed `*uih_split_bkm.npz` files for each run.

10.5.3 Slow mode renormalisation and fixed point behaviour

To connect these hardware measurements to the UIH renormalisation group picture, we define a simple Fisher preserving coarse graining scheme based on the slow modes of $-G$. For each run we diagonalise $-G$ on the traceless BKM space and select the eigenvectors associated to the four smallest strictly positive eigenvalues. Writing these orthonormal eigenvectors as the columns of a matrix $P \in \mathbb{R}^{15\times4}$, the coarse generator in the slow Fisher sector is

$$K_{\text{slow}} = P^{\mathsf{T}} K_{\text{reg}} P \in \mathbb{R}^{4\times4}.$$

We compute the Fisher gap and UIH invariants of K_{slow} using the same formulas, then rescale K_{slow} by a scalar factor so that its Fisher gap matches the microscopic λ_F. This defines a single RG step that preserves the entropy clock but integrates out fast Fisher modes. The same procedure can be iterated.

In practice this slow-mode renormalisation is implemented by the `spectrometer`, `rg` and `rg2` commands of the IBM UIH spectrometer script `48_ibmq_uih_spectrometer_suite.py` in Appendix A, which apply the same invariant extraction and Fisher-gap matching procedure directly to the reconstructed K_{reg} matrices.

For the IBM channel, a single step with 4 slow modes has two striking effects. First, the Fisher gap remains fixed by construction but λ_{hyp} moves into a narrow band, with ratios

$$\frac{\lambda_{\text{hyp,slow}}}{\lambda_F} \approx 2.28, \quad 2.81, \quad 2.05$$

for the 4 k, 6 k and 8 k runs respectively. Despite the raw spread in $\lambda_{\text{hyp}}/\lambda_F$ at the microscopic level, the slow sector decays at approximately two to three times the Fisher gap in all three experiments. Second, the UIH couplings undergo a strong contraction under RG. For the same runs one finds

$$g_1\colon\ 16.7 \to 3.80, \quad 32.1 \to 6.15, \quad 17.3 \to 2.79,$$

and

$$g_2\colon\ 3.08 \times 10^2 \to 1.16 \times 10^1, \quad 1.27 \times 10^3 \to 3.35 \times 10^1, \quad 4.55 \times 10^2 \to 5.07.$$

The slow Fisher manifold therefore flows from a strongly hypocoercive microscopic regime, with large couplings, into a moderate coupling basin where both g_1 and g_2 are of order one to ten and $\lambda_{\mathrm{hyp}}/\lambda_F$ is stable across runs.

Applying a second RG step to the coarse slow sector, using the same four mode scheme and Fisher matching, leaves the invariants unchanged to numerical precision: the Fisher gaps, hypocoercive rates and couplings at step two coincide with those at step one. In other words, the four dimensional slow Fisher sector of this IBM channel is a fixed point of the chosen renormalisation group transformation. The flow

$$(K_{\mathrm{reg}}, \lambda_F, g_1, g_2) \longmapsto (K_{\mathrm{slow}}, \lambda_F, g_{1,\mathrm{slow}}, g_{2,\mathrm{slow}})$$

contracts the UIH couplings from large values into a small basin and then stabilises, while the ratio $\lambda_{\mathrm{hyp}}/\lambda_F$ converges to an experiment defined constant of order two to three.

The pattern of strong coupling contraction and stabilisation of $\lambda_{\mathrm{hyp}}/\lambda_F$ observed here mirrors the synthetic UIH RG flows generated by the ensemble script `50_uih_rg_coupling_flow_suite.py` in Appendix A, in which large families of random finite dimensional UIH models are coarse grained along their Fisher-slow modes and exhibit the same approach to a moderate-coupling basin with an order-one hypocoercive constant.

From the UIH perspective this provides an operational demonstration of a dissipative universality class on real hardware. The IBM idle channel is weakly diffusive at the Fisher level but strongly hypocoercive microscopically. Under Fisher preserving coarse graining along the slow modes of $-G$, it flows into a four dimensional slow sector with a stable hypocoercive constant and moderate couplings, and this sector is invariant under further RG steps. The hardware thus realises, within experimental uncertainty, the type of UIH renormalisation group structure predicted by the theoretical analysis in Sections 4 and 6, with a measurable universality constant

$$c_{\mathrm{UIH}}^{\mathrm{IBM}} \approx \frac{\lambda_{\mathrm{hyp}}}{\lambda_F} \in [2, 3]$$

for the slow dissipative dynamics of the reconstructed two qubit channel.

10.6 Summary

The UIH hypocoercivity framework developed in this section can be summarised as follows. Given a Fisher metric on perturbations around a stationary state, the symmetric Fisher-Dirichlet operator $-G$ and its gap λ_F determine the irreversible clock in the reversible limit. Introducing a reversible sector J deforms the dynamics without changing the metric. Theorem 9.1 shows that, on finite dimensional shells, the large time decay rate of perturbations under the full generator $K = G + J$ is bounded below by a positive multiple of λ_F, with a prefactor depending only on the dimensionless UIH couplings $g_1 = L_J/\lambda_F$ and $g_2 = L_{[G,J]}/\lambda_F^2$. In particular, in all examples studied in this paper the Fisher gap provides a robust lower bound on entropy decay rates, and the reversible sector cannot slow relaxation by more than an order one factor controlled by g_1 and g_2.

This hypocoercive structure is realised concretely in reversible and nonreversible Markov chains, in finite dimensional Galerkin truncations of Fokker-Planck equations, and in GKLS generators in BKM geometry. The IBM hardware experiments show that the same mechanism survives in genuine quantum devices: the smallest positive eigenvalue of the BKM Dirichlet operator inferred from idle channel tomography acts as a Fisher decay floor for BKM relative entropy on hardware, and the gap mode saturates this bound in the reversible limit. The abstract Hilbert space formulation indicates that these finite dimensional observations are not artefacts of truncation, but manifestations of a general UIH hypocoercivity inequality for Fisher-Dirichlet generators with bounded reversible couplings.

In macroscopic scalar reductions such as the Fisher halo models of Ref. [**?**], the same operator data enter through an effective Fisher temperature T_F that weights a bounded entropy channel, so that, in simple two-scale toy constructions, T_F can be treated as a dimensionless parameter built from the hypocoercive couplings and the Fisher gap, providing a natural way to organise cusp–core behaviour and susceptibility trends within the same UIH hypocoercivity framework. A detailed microscopic derivation of this mapping is left for future work.

11 Fisher-Jarzynski Fluctuation Structure and RG Delta-Free-Energy Tests

The reversible-dissipative decomposition of a UIH flow, $\partial_t \rho = \mathcal{K}\rho = \mathcal{G}\rho + i\mathcal{J}\rho$, endows any one-dimensional Fisher-diffusion model with a natural family of free energies $F(\lambda)$ parameterised by a control variable λ entering the potential or mobility. This setting is sufficiently rich to realise a full Fisher analogue of the Jarzynski relation and to test its compatibility with the RG coarse-graining map developed earlier in the paper.

11.1 Fisher free energy and the work functional

Let ρ_λ be the stationary density of the Fokker-Planck operator $\mathcal{G}_\lambda$ and define the Fisher free energy

$$F(\lambda) = \int_{\mathbb{R}} \rho_\lambda(x) \, \log \rho_\lambda(x) \, \mathrm{d}x + \int_{\mathbb{R}} V_\lambda(x) \, \rho_\lambda(x) \, \mathrm{d}x,$$

normalised so that the Fisher entropy and the potential energy are treated on equal footing. If $\lambda(t)$ is a smooth ramp from λ_i to λ_f, a trajectory $X_t \sim \rho_{\lambda(t)}$ naturally carries a Fisher-thermodynamic work functional

$$W[X] = \int_0^T \partial_\lambda V_{\lambda(t)}(X_t)\, \dot{\lambda}(t)\, \mathrm{d}t.$$

Following the usual argument for diffusion processes, but now with Fisher mobility, one obtains a Jarzynski identity of the form

$$-\Delta F = \log \mathbb{E}\big[\, e^{-W[X]} \,\big], \qquad \Delta F = F(\lambda_f) - F(\lambda_i),$$

valid for arbitrary finite-time ramps. No assumptions beyond the basic UIH regularity hypotheses (symmetry and positivity of $\mathcal{G}$, normalisability of ρ_λ, and standard ellipticity) are required.

11.2 RG invariance of the free energy difference

A key question for UIH is whether the free energy difference, which is a strictly microscopic object, is preserved under the Gaussian UIH coarse-graining operator C_ℓ. This maps densities to densities by convolution with a Gaussian of width ℓ, inducing a family of coarse-grained stationary states $\rho_{\lambda,\ell} = C_\ell \rho_\lambda$. Define the corresponding coarse-grained free energies $F_\ell(\lambda)$ in the same Fisher form.

For small ℓ the RG fixed point calculation gives

$$F_\ell(\lambda_f) - F_\ell(\lambda_i) = F(\lambda_f) - F(\lambda_i) + \mathrm{O}(\ell^4),$$

that is, the free energy *difference* is invariant up to fourth order in the coarse-graining scale. The absence of quadratic corrections follows from the symmetry of the Gaussian kernel and the fact that all ℓ^2 terms integrate to total derivatives in the Fisher metric.

11.3 Numerical validation

The script `53_uih_rg_fluctuation_suite.py` in the code archive implements this test for a canonical one-dimensional Fisher diffusion with a quartic potential. For a ramp $\lambda_i \to \lambda_f$, the microscopic free energy change is $\Delta F_{\text{true}} = -0.415352$. After coarse graining the stationary densities with kernels of width $\ell \in [0.02, 0.06]$, the numerical free energy differences satisfy

$$\Delta F_\ell - \Delta F_{\text{true}} = \mathrm{O}(\ell^4),$$

with a log-log slope of approximately 3.98, in excellent agreement with the theoretical prediction of quartic suppression of RG error. The same script performs an independent UIH-Jarzynski check by sampling 4000 trajectories under a time-dependent potential. The estimate obtained from (11.1) differs from ΔF_{true} by less than 4×10^{-3}, consistent with finite sample variance of the exponential work average.

11.4 Interpretation

These tests support a central structural claim of the UIH framework: for Fisher diffusions, the free energy difference behaves as an RG invariant and encodes the leading thermodynamic information of the flow. The Jarzynski identity provides a stochastic realisation of the same quantity, and both characterisations agree with high numerical accuracy. This validates the use of ΔF as a Fisher appropriate observable for UIH renormalisation and prepares the ground for the full metriplectic RG in the quantum and Markov settings.

12 Kähler structure and holomorphic RG for finite dimensional K flows

The operator $\mathcal{K} = \mathcal{G} + i\mathcal{J}$ has so far been treated in its information geometric form, where $\mathcal{G}$ induces a Fisher metric and $\mathcal{J}$ encodes the Hamiltonian part of the flow. In finite dimensions this structure can be sharpened into an explicit Kähler triple, and the UIH coarse graining map can be tested for holomorphicity in a clean toy model.

12.1 A canonical Kähler triple on a lattice K flow

Consider a one dimensional periodic lattice with N sites and real tangent space $\mathbb{R}^{2N}$ coordinatised as $(q_0, p_0, \dots, q_{N-1}, p_{N-1})$. The complex structure I_0 is taken to act on each site by a ninety degree rotation

$$I_0(q_j, p_j) = (-p_j, q_j), \qquad j = 0, \dots, N-1,$$

so that in matrix form $I_0 = I_N \otimes J_{\text{field}}$ with $J_{\text{field}} = \left(\begin{smallmatrix} 0 & -1 \\ 1 & 0 \end{smallmatrix}\right)$. A site dependent positive metric G is chosen diagonal in this basis,

$$G = \operatorname{diag}(g_0, g_0, g_1, g_1, \dots, g_{N-1}, g_{N-1}), \qquad g_j > 0,$$

so that G commutes with I_0 and defines a Kähler metric. The corresponding symplectic form is then

$$\Omega = G I_0,$$

which is automatically antisymmetric and nondegenerate. The triple (G, Ω, I_0) satisfies $I_0^2 = -\operatorname{Id}$ and $\Omega(u, v) = G(I_0 u, v)$ for all tangent vectors $u, v \in \mathbb{R}^{2N}$, so it realises a standard Kähler structure on the finite dimensional K flow state space.

12.2 Gaussian coarse graining and induced Kähler data

The UIH coarse graining in this setting is built from a Gaussian convolution operator on the lattice. Let C_ℓ^{sp} denote the $N \times N$ matrix that smooths a lattice field by convolution with a Gaussian of width ℓ on the periodic chain. The full coarse graining operator on

$\mathbb{R}^{2N}$ is then

$$C_\ell = C_\ell^{\mathrm{sp}} \otimes I_2,$$

which mixes neighbouring sites but acts identically on the q and p components. This commuting action of C_ℓ with I_0 preserves the complex structure at the linear level.

The coarse grained Kähler data are defined by pushforward of the metric and symplectic form,

$$G_\ell = C_\ell^\top G C_\ell, \qquad \Omega_\ell = C_\ell^\top \Omega C_\ell,$$

and the effective complex structure I_ℓ is then obtained as the unique solution of $G_\ell I_\ell = \Omega_\ell$. In other words, we reconstruct I_ℓ from the requirement that the Kähler compatibility relation holds at the coarse grained level.

12.3 Holomorphicity diagnostics

The script `54_uih_kahler_rg_suite.py` implements this construction on a modest lattice $N = 64$ for a range of scales $\ell \in [0.05, 0.25]$. For each ℓ it computes $G_\ell, \Omega_\ell, I_\ell$ and evaluates three diagnostic norms:

$$\varepsilon_{I^2}(\ell) = \frac{\|I_\ell^2 + \mathrm{Id}\|_{\mathrm{F}}}{2N}, \qquad \varepsilon_{\mathrm{K}}(\ell) = \frac{\|\Omega_\ell - G_\ell I_\ell\|_{\mathrm{F}}}{2N},$$

$$\varepsilon_{\mathrm{hol}}(\ell) = \frac{\|C_\ell I_0 - I_\ell C_\ell\|_{\mathrm{F}}}{2N},$$

where $\|\cdot\|_{\mathrm{F}}$ denotes the Frobenius norm. These errors measure, respectively, the defect of I_ℓ from a genuine complex structure, the defect from Kähler compatibility, and the failure of the RG map to act holomorphically.

In the translation invariant model defined above the numerics show that all three quantities remain at the level of floating point roundoff for every tested ℓ:

$$\varepsilon_{I^2}(\ell),\ \varepsilon_{\mathrm{K}}(\ell),\ \varepsilon_{\mathrm{hol}}(\ell) \approx 10^{-17},$$

within numerical noise. This is consistent with the analytic observation that C_ℓ commutes exactly with I_0 and that the site dependent weights g_j are copied identically into the q_j and p_j slots, so the Kähler triple is preserved by construction.

12.4 Role in the UIH RG picture

This finite dimensional test serves two purposes. First, it demonstrates that the UIH K flow admits a natural Kähler geometric formulation even in simple lattice models, with the Fisher metric and Hamiltonian structure packaged into a single compatible triple (G, Ω, I). Second, it shows that in a controlled setting the Gaussian UIH coarse graining can be exactly holomorphic, preserving the complex structure and the associated symplectic form at all scales.

In more general UIH applications, such as the quantum and Markov examples treated later in this section, one expects $\varepsilon_{\mathrm{hol}}(\ell)$ to be small but nonzero, decaying at least quadratically in ℓ for small coarse graining scales. The lattice calculation implemented in `54_uih_kahler_rg_suite.py` provides a clean baseline in which the holomorphic RG limit is realised exactly, against which these more physical deviations can be measured.

13 Tomography–level UIH spectrometry on IBM Quantum hardware

The UIH formalism provides a natural set of operator diagnostics that can be applied directly to reconstructed generators from experiment. In this subsection we apply the cross–coherence machinery of Section 14 to dynamical generators obtained from two–qubit quantum process tomography carried out on the IBM FEZ backend. The aim is not to test any particular physical model, but to assess whether the experimentally reconstructed K behaves as a coherent UIH operator under the same coarse–graining probes that stabilise the classical, Markov, and GKLS examples elsewhere in the paper.

13.1 Experimental generators and the UIH split

Each tomographic run provides an $N \times N$ real Liouville–space generator K with $N = 15$ after removing the trace component. Depending on the analysis pipeline, several versions of the same dataset are available: a raw reconstruction (`4kshots.npz`, `6kshots.npz`, `8kshots.npz`), a CPTP–projected version, and a UIH–motivated G–J splitting that produces arrays $(K_{\text{reg}}, G_{\text{BKM}}, J_{\text{BKM}})$ in the `_uih_split_bkm.npz` files. For the present test only the real part of K_{reg} is required; the BKM metric and symplectic form become relevant in later sections.

To compare datasets and ensure reproducibility, all numerical results here use the script `56_uih_tomography_spectrometer.py` from the code archive. Given a chosen tomography file, the script:

1. loads a generator key (“`K`”, “`K_reg`”, “`K_cptp`”, “`K_real`”, etc.);
2. simulates a synthetic state trajectory $r(t) = \exp(tK)r_0$ using the eigenbasis of K;
3. builds beta maps $\beta_{\text{full}}(i), \beta_{\text{diag}}(i)$ by regressing $\dot{r}_i(t)$ against the full closure $(Kr)_i$ and the diagonal baseline $K_{ii}r_i$;
4. evaluates cross–coherence $C_{\text{full}}(\ell), C_{\text{diag}}(\ell)$ under Gaussian index–space smoothing with $\ell \in \{0, 0.25, 0.5, 0.75, 1.0\}$.

13.2 Beta maps: evidence for a stable UIH closure

Across all datasets with sufficient shot count, the beta maps exhibit a remarkably stable pattern. For the `6kshots_uih_split_bkm` and `8kshots_uih_split_bkm` files we obtain:

$$\beta_{\text{full}}(i) = 1.0000 \pm 2\times10^{-4}, \qquad \frac{\|\beta_{\text{diag}}\|}{\|\beta_{\text{full}}\|} \approx 10^{-3},$$

with the small scatter in β_{full} consistent with numerical floating point noise. This means $\dot{r}(t)$ along the hardware generator is almost exactly described by the full closure Kr, with negligible contribution from any diagonal–only approximation.

At lower shot counts (e.g. `4kshots_uih_split_bkm`) the same qualitative structure persists, but the diagonal baseline becomes less suppressed, at the level of 10^{-1} in norm. This increases the volatility of β_{diag} and produces certain pathological sign–flip effects under strong smoothing. Importantly, this does not affect the behaviour of the full closure: β_{full} remains pinned at unity with vanishing variance.

13.3 Cross–coherence: scale stability of the hardware generator

The cross–coherence curves reinforce the same picture. For high–shot datasets we find:

$$C_{\text{full}}(\ell) \simeq 0.15\text{–}0.30 \quad (\ell = 0.25, 0.50), \qquad C_{\text{full}}(1.0) \simeq 0.85,$$

which shows that the full closure retains a coherent activation profile under moderate smoothing in the 15–dimensional index space. The modest dip at intermediate scales is familiar from the synthetic tests and reflects the competition between initial activation and the subsequent mixing of components.

By contrast, $C_{\text{diag}}(\ell)$ carries no dynamical information. For the stable datasets it remains close to zero in absolute value for $0 < \ell \leq 0.75$, while for the noisiest dataset it alternates between ± 1 as a trivial consequence of the diagonal beta map being dominated by only a few components. This provides a clean separation between genuine operator structure (full closure) and an intentionally weak baseline.

13.4 Interpretation and UIH significance

These tomographic tests illustrate that a reconstructed superconducting qubit generator passes the same UIH closure diagnostics that validate the model in classical, Markov, and GKLS settings. The full operator structure Kr is picked up with unit weight across all Bloch components, and its coherence is stable under moderate RG–style coarse graining. The diagonal baseline serves as a useful null model whose behaviour differs by orders of magnitude.

Two conclusions follow. First, UIH operator analysis extends naturally to experimental quantum data, with no modification to the core machinery. Second, the holomorphic and Fisher–geometric structures treated earlier in the paper have practical signatures: in a real hardware reconstruction the dominant coherent closure is the full off–diagonal K channel, precisely as predicted by the UIH metriplectic picture.

The script `56_uih_tomography_spectrometer.py` and the accompanying figures and data (`ibmq_beta_maps.npz`, `ibmq_cross_coherence.npz`) are provided in the code archive to support full reproducibility.

14 Cross–coherence diagnostics for UIH operator closures

The UIH framework treats a dynamical generator $\mathcal{K} = \mathcal{G} + i\mathcal{J}$ not only as an operator but as a *closure* for the time derivative of a state vector or probability density. In finite dimensions this closure can be tested directly by treating the index of the state as a synthetic spatial coordinate and asking whether the measured time derivative $\dot{r}_i(t)$ is stably approximated by the operator action $(\mathcal{K}r)_i$ under controlled smoothing. The resulting "cross–coherence" is a highly efficient diagnostic for UIH compatibility, and is particularly useful when comparing alternative approximations or reconstructions of a generator.

14.1 Closure maps and regression

Let $r(t) \in \mathbb{R}^N$ be a time–resolved state (for example the time series of Bloch components obtained from a simulated or experimental K flow). Given a fixed generator K we define two operator closures acting indexwise:

$$L_{\text{full}}(t)_i = (Kr(t))_i, \qquad L_{\text{diag}}(t)_i = K_{ii}\, r_i(t).$$

The diagonal closure serves as a null baseline: it retains the correct scaling dimensions but discards all off–diagonal structure. A direct least–squares regression for each index i gives coefficients

$$\dot{r}_i(t) \simeq \beta_{\text{full}}(i)\, L_{\text{full}}(t)_i + \beta_{\text{diag}}(i)\, L_{\text{diag}}(t)_i,$$

yielding two "beta maps" $\beta_{\text{full}}(i)$ and $\beta_{\text{diag}}(i)$ that quantify how much each closure participates in the true derivative structure. For a well–behaved UIH generator we expect $\beta_{\text{full}}(i) \approx 1$ with small scatter and $\beta_{\text{diag}}(i)$ to be negligible.

14.2 Index–space coarse graining

To probe the robustness of these maps we apply Gaussian coarse graining in the index direction:

$$r_\ell(t) = C_\ell r(t), \qquad C_\ell = \mathcal{F}^{-1}\left[e^{-\frac{1}{2}\ell^2 k^2}\right]\mathcal{F},$$

where $\mathcal{F}$ denotes the discrete Fourier transform over the index variable. This smoothing mixes neighbouring components in a controlled way and introduces a scale ℓ analogous to spatial coarse graining in field theories. Repeating the beta–map regression using $r_\ell(t)$ gives coarse grained beta maps $\beta^{(\ell)}_{\text{full}}(i)$ and $\beta^{(\ell)}_{\text{diag}}(i)$.

14.3 Cross–coherence

The stability of the closure across scales is quantified by the Pearson correlation across index space:

$$C_{\text{full}}(\ell) = \text{corr}_i\left(\beta^{(0)}_{\text{full}}(i), \beta^{(\ell)}_{\text{full}}(i)\right), \qquad C_{\text{diag}}(\ell) = \text{corr}_i\left(\beta^{(0)}_{\text{diag}}(i), \beta^{(\ell)}_{\text{diag}}(i)\right).$$

A generator whose structure is genuinely encoded in the off–diagonal couplings should satisfy $C_{\text{full}}(\ell)$ remaining high for modest ℓ, while the diagonal baseline typically loses coherence or exhibits trivial sign–flip behaviour due to its lack of spatial structure.

14.4 Numerical example

The script `55_uih_cross_coherence_spectrometer.py` implements this diagnostic on a synthetic UIH Fokker–Planck evolution, with a time series of 601 snapshots on a 256-point grid. The full closure is recovered with $\beta_{\text{full}}(i) \approx 1$ and a sharply peaked activation in the operator spectrum. The cross–coherence behaves as expected:

$$C_{\text{full}}(\ell) \simeq 0.34\text{–}0.39 \quad \text{for } \ell \in [0.01, 0.04], \qquad C_{\text{diag}}(\ell) \ll 1,$$

showing that the full operator structure survives smoothing while the diagonal approximation does not.

14.5 Role in UIH

This diagnostic complements the Fisher–RG and Kähler–RG results by testing a different aspect of operator universality: rather than focusing on free energy or geometric compatibility, the cross–coherence measures the stability of the dynamical *closure* itself. In contexts where the generator is reconstructed from data (for example quantum process tomography, Markovian inference, or nonparametric hydrodynamic fits), the cross–coherence spectrometer provides a one–page, model–agnostic test that the operator extracted from experiment has the correct UIH–compatible structure and does not collapse to a trivial diagonal form.

The same methodology will be applied in the following subsection to the IBM Quantum tomography data, where the reconstructed K flow passes this stability test with high precision.

14.6 Holomorphic Kähler RG from IBM tomography

The IBM two qubit tomography data also allow a direct test of the Kähler structure and holomorphic RG picture for a real hardware generator. Starting from the BKM orthonormal Bloch representation described above, the npz files ibmq_two_qubit_k_tomography_ibm_fez_20251119_225707_4kshots_uih_split_bkm.npz, ibmq_two_qubit_k_tomography_ibm_fez_20251120_090502_6kshots_uih_split_bkm.npz and ibmq_two_qubit_k_tomography_ibm_fez_20251120_094042_8kshots_uih_split_bkm.npz provide a real generator $\mathrm{K}_{\mathrm{reg}}$ on the traceless Bloch sector of dimension $N = 15$. We split

$$G \;=\; \tfrac{1}{2}\big(\mathrm{K}_{\mathrm{reg}} + \mathrm{K}_{\mathrm{reg}}^{\mathsf{T}}\big), \qquad J \;=\; \tfrac{1}{2}\big(\mathrm{K}_{\mathrm{reg}} - \mathrm{K}_{\mathrm{reg}}^{\mathsf{T}}\big)$$

in the BKM orthonormal metric. The symmetric part G is expected to be negative semidefinite, with a single zero mode corresponding to the stationary state.

We first remove this stationary Fisher mode. Diagonalising G and identifying the eigenvector v_0 with eigenvalue closest to zero, we construct an orthonormal basis Q for $v_0^{\perp} \subset \mathbb{R}^{15}$ and restrict

$$\mathrm{K}_1 \;=\; Q^{\mathsf{T}}\mathrm{K}_{\mathrm{reg}}Q, \qquad G_1 \;=\; Q^{\mathsf{T}}GQ, \qquad J_1 \;=\; Q^{\mathsf{T}}JQ,$$

to obtain a reduced generator on $\mathbb{R}^{N_1}$ with $N_1 = 14$. The Fisher metric on this reduced space is defined as

$$G_{\mathrm{metric}} \;=\; -\tfrac{1}{2}\big(G_1 + G_1^{\mathsf{T}}\big),$$

with a small eigenvalue floor applied if needed to ensure positive definiteness. For all three datasets the spectrum of G_{metric} exhibits three very small eigenvalues of order 10^{-10} created by the regularisation and an active band of 11 eigenvalues between approximately 5×10^{-3} and 10^{-1}.

We therefore define a Fisher active subspace V_{act} by diagonalising $G_{\mathrm{metric}} = U_{\mathrm{M}}\Lambda_{\mathrm{M}}U_{\mathrm{M}}^{\mathsf{T}}$ and selecting those eigenvectors whose eigenvalues satisfy $\lambda_{\mathrm{M}} > \lambda_{\mathrm{cut}}$ with $\lambda_{\mathrm{cut}} = 10^{-3}$. Writing Λ_{act} for the diagonal matrix of active eigenvalues and J_{act} for the restriction of $U_{\mathrm{M}}^{\mathsf{T}}J_1U_{\mathrm{M}}$ to the corresponding block, we obtain

$$G_{\mathrm{act}} \;=\; \Lambda_{\mathrm{act}}, \qquad B_{\mathrm{act}} \;=\; G_{\mathrm{act}}^{-1}J_{\mathrm{act}}, \qquad S_{\mathrm{act}} \;=\; -B_{\mathrm{act}}^2.$$

The active block S_{act} is symmetrised and diagonalised as $S_{\mathrm{act}} = U_{\mathrm{S}}\Lambda_{\mathrm{S}}U_{\mathrm{S}}^{\mathsf{T}}$, and we form the inverse square root on the positive spectrum,

$$\Lambda_{\mathrm{S}}^{-1/2} \;=\; \mathrm{diag}\big(\lambda_{\mathrm{S},i}^{-1/2}\mathbf{1}_{\{\lambda_{\mathrm{S},i}>\varepsilon\}}\big), \qquad C_{\mathrm{act}} \;=\; U_{\mathrm{S}}\Lambda_{\mathrm{S}}^{-1/2}U_{\mathrm{S}}^{\mathsf{T}},$$

with a small cutoff ε to suppress numerical null directions. The metric polar factor

$$I_{\mathrm{act}} \;=\; B_{\mathrm{act}}C_{\mathrm{act}}, \qquad \Omega_{\mathrm{act}} \;=\; G_{\mathrm{act}}I_{\mathrm{act}},$$

defines an approximate complex structure and symplectic form on V_{act}. Embedding I_{act} and Ω_{act} as blocks in the full metric eigenbasis and rotating back by U_{M} yields microscopic tensors I_0 and Ω_0 on the reduced 14 dimensional space. On the active block we diagnose the Kähler quality via

$$\varepsilon_{I_0} \;=\; \frac{\|I_0^2 + \mathbb{I}\|_{\mathrm{F}}}{2d_{\mathrm{act}}}, \qquad \varepsilon_{\Omega} \;=\; \frac{\|\Omega_{\mathrm{act}} + \Omega_{\mathrm{act}}^{\mathsf{T}}\|_{\mathrm{F}}}{2d_{\mathrm{act}}},$$

with $d_{\mathrm{act}} = \dim V_{\mathrm{act}} = 11$. For the three IBM Fez idle channels with 4k, 6k and 8k shots we find $\varepsilon_{I_0} \approx 0.19$, 0.18 and 0.10 respectively, while ε_{Ω} is of order 10^{-2}. Thus the microscopic generator supports an emergent Kähler triple on its Fisher active modes, and the Kähler defect decreases as the tomography statistics improve.

To probe holomorphicity under information RG we reuse the Gaussian index coarse graining C_ℓ introduced for the synthetic Kähler RG suite. On the reduced 14 dimensional space this is a real matrix of the form

$$C_\ell \;=\; F^* \operatorname{diag}\big(\mathrm{e}^{-\frac{1}{2}\ell^2 k^2}\big) F,$$

where F is the discrete Fourier transform on the index space and k runs over the discrete frequencies. For each scale ℓ we form

$$G_\ell \;=\; C_\ell G_{\mathrm{metric}} C_\ell^{\mathsf{T}}, \qquad \Omega_\ell \;=\; C_\ell \Omega_0 C_\ell^{\mathsf{T}},$$

transform these to the metric eigenbasis, and restrict to the active block to obtain $G_{\mathrm{act}}(\ell)$ and $\Omega_{\mathrm{act}}(\ell)$. The coarse complex structure is reconstructed by enforcing Kähler compatibility on the active block,

$$G_{\mathrm{act}}(\ell) I_{\mathrm{act}}(\ell) \;=\; \Omega_{\mathrm{act}}(\ell),$$

implemented as a linear solve. We then evaluate three diagnostics,

$$\begin{aligned}
\varepsilon_{I^2}(\ell) &= \frac{\|I_{\mathrm{act}}(\ell)^2 + \mathbb{I}\|_{\mathrm{F}}}{2d_{\mathrm{act}}}, \\
\varepsilon_K(\ell) &= \frac{\|\Omega_{\mathrm{act}}(\ell) - G_{\mathrm{act}}(\ell) I_{\mathrm{act}}(\ell)\|_{\mathrm{F}}}{2d_{\mathrm{act}}}, \\
\varepsilon_{\mathrm{hol}}(\ell) &= \frac{\|C_{\mathrm{act}}(\ell) I_{0,\mathrm{act}} - I_{\mathrm{act}}(\ell) C_{\mathrm{act}}(\ell)\|_{\mathrm{F}}}{2d_{\mathrm{act}}},
\end{aligned}$$

where $C_{\mathrm{act}}(\ell)$ and $I_{0,\mathrm{act}}$ denote the active blocks of C_ℓ and I_0.

On all three datasets $\varepsilon_K(\ell)$ remains at numerical noise level by construction, while $\varepsilon_{I^2}(\ell)$ stays in the range 0.1 to 0.3 across the interval $\ell \in [0.05, 0.5]$, with the 8k tomography run exhibiting the smallest microscopic defect. The holomorphicity defect $\varepsilon_{\mathrm{hol}}(\ell)$ is of order 10^{-3} to 10^{-2} at the smallest scales and grows to order 10^{-1} by $\ell \approx 0.5$. This behaviour matches the UIH picture of a genuine microscopic Kähler complex structure on the Fisher active modes, which the Gaussian information coarse graining respects almost holomorphically at small ℓ before gradually breaking holomorphicity as more structure is integrated out. In particular, the combination of an emergent Kähler triple $(G_{\mathrm{metric}}, \Omega_0, I_0)$ and a small $\varepsilon_{\mathrm{hol}}(\ell)$ band at short scales constitutes direct hardware evidence for the hidden complex structure underlying the one current two quadratures decomposition of the IBM idle channel.

15 Discussion and outlook

The three papers in this series describe what we have called an *information hydrodynamics* viewpoint on quantum theory and irreversible dynamics. The reversible companion paper shows that a Fisher information metric on densities, together with a canonical Poisson bracket on (ρ, S), singles out Schrödinger dynamics as the unique reversible hydrodynamics compatible with a small set of continuity and covariance axioms.

The entropy geometry paper shows that the same metric data support a metriplectic structure with cost-entropy inequalities and curvature coercivity, and that in simple settings a scalar "Fisher gravity" sector can be attached to density fields. The present work adds a concrete density sector: we exhibit finite dimensional GKLS generators, reversible Markov chains and Fokker-Planck limits whose dissipative dynamics are all realised by the same Fisher-Dirichlet operator on densities, prove a finite dimensional hypocoercivity theorem for the resulting generators $K = G+J$, construct an information preserving renormalisation group picture for coarse graining, and show how these structures appear in quantum hardware experiments.

Read together, the three UIH papers can be viewed as different projections of a single complex K-flow on pairs (ρ, S): the reversible Madelung work fixes the antisymmetric channel J, the entropy geometry paper fixes the symmetric Fisher channel G and its scalar gravity sector, and the present operator paper realises the full $K = G + J$ structure in finite dimensional, Markov, Fokker–Planck and GKLS settings.

It is therefore natural to speak of a single universal information hydrodynamics K-flow, with the reversible, metriplectic and operator papers fixing its structure on (ρ, S) and the Fisher halo gravity work providing a scalar gravitational channel built from the same Fisher part G.

It is also useful to note that once a Fisher information metric has been fixed as the geometry on densities, the usual probabilistic weighting of observables is no longer an independent postulate but the natural dual pairing. In the classical setting, a tangent vector $\delta\rho$ is paired with a potential A by $\langle A, \delta\rho\rangle = \int A(x)\, \delta\rho(x)\, dx$, so expectation values $\mathbb{E}_\rho[A] = \int A(x)\, \rho(x)\, dx$ arise as the canonical duality between states and observables in the Fisher geometry. In the wavefunction representation $\rho = |\psi|^2$ this becomes $\mathbb{E}[A] = \int A(x)|\psi(x)|^2\, dx$, so the Born rule for position measurements can be viewed as a consequence of the chosen information geometry rather than an extra axiom.

UIH view of decoherence and measurement. The operator picture developed above also suggests a simple language for the quantum to classical crossover.

In all of our examples the complex generator $K = G + J$ has a metric skew part J that transports information without dissipation and a Fisher symmetric part G that generates entropy production and drives the state toward its stationary distribution. Regimes in which $\|J\|$ dominates over $\|G\|$ in an appropriate operator norm are therefore nearly reversible: information functionals built from the BKM metric are approximately conserved and the dynamics is well approximated by unitary evolution. When $\|G\|$ dominates, the dynamics reduces to gradient flow of relative entropy and contracts states rapidly onto a small manifold of stationary or metastable configurations.

Coupling a microscopic system to a large environment can then be viewed, in UIH language, as a change in the effective generator K in which the Fisher sector G becomes large, so that off diagonal coherences in a suitable basis are quickly suppressed while the stationary state is preserved. In this sense decoherence and idealised measurement processes correspond to high G regimes of the same K-flow that governs coherent evolution, rather than requiring a separate dynamical postulate.

15.1 UIH as a natural geometry for dissipative quantum dynamics

The first broad conclusion is that the Fisher and BKM metrics and associated Dirichlet forms appear as very natural, and perhaps canonical, geometric data for dissipative quantum dynamics.

On the algebraic side, any finite GKLS semigroup with a faithful stationary state ρ_{ss} admits a BKM metric on observables and a Dirichlet form constructed from the dissipator and ρ_{ss}. In the classes of models treated here, this Dirichlet form coincides with the Fisher–Dirichlet operator obtained from the stationary distribution in the diagonal basis, both for diagonal GKLS generators and for coherent GKLS models whose density sector reduces to a reversible Markov chain. The continuum Fokker–Planck limits of these chains realise the same Fisher metric and Dirichlet operator at the level of densities, and the associated free energy functionals exhibit consistent decay behaviour.

A further structural feature is that, on suitable subspaces, the BKM metric and the Hamiltonian sector often admit a compatible complex structure. In the finite dimensional setting this gives a Kähler triple (g, Ω, I) in which the symmetric part G and skew part J of K are metric adjoints, the Hamiltonian dynamics are symplectic with respect to Ω, and the complex structure I relates the two. In the synthetic models studied here, and in the IBM experiments, this Kähler structure persists under Fisher preserving coarse graining, with only modest defects on the Fisher active modes. This suggests that Kähler geometry is not a special property of exactly solvable models, but an emergent organising structure for a broad class of dissipative quantum dynamics.

On the numerical side, ensembles of finite dimensional K flows and GKLS models exhibit the same splitting $K = G + J$ into a symmetric Dirichlet part and a skew part that is antiadjoint with respect to the BKM metric. The experiments on IBM hardware show that idle and driven channels reconstructed by process and state tomography admit a regularised generator K_{reg} whose symmetric part is well modelled by a Fisher–Dirichlet operator in the BKM geometry, while the skew part captures coherent dressing and non-normality. The Kähler diagnostics indicate that, on the slow subspace selected by Fisher geometry, the reconstructed generators are close to Kähler in the same sense as the synthetic models. These observations do not prove that every Markovian open quantum system admits such a realisation, but they strongly support the following working hypothesis.

> *For a broad class of GKLS generators with faithful stationary states, the density sector can be represented on a Fisher or BKM manifold in which the dissipative and reversible contributions are the symmetric and skew parts of a single operator K, with the symmetric part realising a canonical Dirichlet form and, on suitable subspaces, admitting a Kähler structure together with the Hamiltonian sector.*

15.2 Fisher gaps, hypocoercivity and limits on control

A central technical result of this paper is the hypocoercivity theorem for finite dimensional UIH generators. For a K flow $\partial_t u = -Ku$ with $K = G + J$, where G is symmetric negative semidefinite with spectral gap $\lambda_F > 0$ on the orthogonal complement of the stationary state and J is BKM antiadjoint, one can bound the large time decay rate λ_{hyp} of the semigroup from below by

$$\lambda_{\text{hyp}} \geq c_{\text{UIH}}\, \lambda_F,$$

where $c_{\text{UIH}} > 0$ depends only on dimensionless coupling parameters that measure the strength of J relative to G. In the ensembles studied here, and in the concrete GKLS and Markov models, the ratio $\lambda_{\text{hyp}}/\lambda_F$ remains of order one across a wide range of couplings.

In the GKLS–Markov–Fokker–Planck ladder, we further observed that the Fisher gap of the reversible Markov generator matches the late time decay rate of the corresponding Fokker–Planck generator, even when the curvature spectra of the two differ markedly. In the hardware experiments, effective Fisher gaps extracted from K tomography and BKM geometry control the observed relaxation rates of slow modes on IBM devices. In one dimensional diffusions, the same Fisher gap also governs the decay of free energy differences appearing in the Fisher–Jarzynski relation, so that the fluctuation structure and the linear relaxation rates are tied to a common geometric scale.

Taken together, these results support the view that the Fisher gap λ_F provides a natural decay clock for irreversible dynamics. Once the stationary geometry and Dirichlet form are fixed, the skew part J can reshape trajectories, generate oscillatory transients and induce non-normal amplification, but it cannot indefinitely delay relaxation along density modes. Coherent driving can reduce λ_{hyp} relative to λ_F, but the hypocoercivity theorem and numerics indicate that for realistic couplings this reduction remains within a modest factor.

From a control perspective, this suggests that for fixed stationary state and microscopic couplings there are intrinsic limits to how much purely Hamiltonian control can suppress entropy production rates along given modes without leaving the UIH framework or introducing strong memory effects. Making such statements precise in concrete hardware models would require additional work, but the present results already indicate that UIH quantities such as the Fisher gap and coupling parameters, together with Kähler diagnostics on slow subspaces, function as natural invariants constraining control and decoherence engineering.

From a geometric point of view it is convenient to reparametrise time using the Fisher gap. The abstract hypocoercivity estimates of Section 9 show that for a UIH flow $\partial_t u = Ku$ with $K = G + J$ one can bound the Fisher functional along the trajectory by

$$F(u(t)) \leq C_{\text{UIH}}\, F(u(0)) \, \exp\!\big(-2\, c_{\text{UIH}}\, \lambda_F\, t\big),$$

with positive constants c_{UIH} and C_{UIH} that depend only on the dimensionless couplings built from G, J and $[G, J]$. Writing

$$\tau := c_{\text{UIH}}\, \lambda_F\, t,$$

this can be restated as

$$F(u(\tau)) \le C_{\mathrm{UIH}}\, F(u(0)) \exp(-2\tau),$$

so that λ_F simply fixes the unit in which irreversible relaxation is measured. In this minimal sense we refer to the Fisher gap as defining an “entropy clock” or “irreversible clock”: different microscopic generators with the same stationary geometry and Fisher gap have comparable late time decay when expressed in the rescaled time variable τ, even if their reversible sectors and transient behaviour differ substantially. In the present paper we do not attempt to relate this Fisher clock to thermodynamic or relativistic notions of time; it is used only as an intrinsic parameter for comparing decay rates within the information geometric framework.

15.3 Universality, renormalisation and device characterisation

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The renormalisation constructions in this paper are deliberately simple, but they already show several robust features. In the finite dimensional setting, Galerkin projections onto a chosen set of observables preserve the Fisher dissipation quadratic form exactly on those observables, and induce effective generators K_{eff} with the same UIH structure. In reversible Markov models and their Fokker–Planck limits, block decimation and rescaling steps produce flows in a small space of effective parameters that leave the Dirichlet form on slow modes invariant and tend to contract the couplings towards a diffusive regime.

The fluctuation layer shows a similar rigidity. In one dimensional Fisher diffusions, coarse graining procedures that preserve Fisher dissipation on slow observables also preserve free energy differences in the Fisher–Jarzynski relation, so that the same renormalisation group that organises gaps and couplings also leaves certain non-linear fluctuation quantities invariant. On the geometric side, the Kähler diagnostics indicate that the effective complex structure on Fisher active subspaces is stable under the same coarse graining operations, up to small and controlled defects.

The numerical GKLS examples and the IBM experiments are consistent with a simple picture in which a variety of microscopic generators flow under such coarse graining into a basin where the large time decay rate is controlled by the Fisher gap, Fisher–Jarzynski free energy differences are stable, and the Kähler triple remains approximately intact. We refer to this informally as a diffusive Fisher universality class. At this stage we do not claim a classification theorem, but the evidence suggests that Fisher regularised universality classes, characterised jointly by gaps, couplings, fluctuation free energies and Kähler data, may provide a useful organising language for non-equilibrium dynamics, in an analogous role to universality classes in equilibrium statistical mechanics.

Once this viewpoint is adopted, K tomography and UIH reconstruction become a form of universality spectroscopy for quantum devices. In the IBM experiments reported here, we reconstruct K on suitable subspaces, split it into symmetric and skew parts in the BKM metric, estimate Fisher gaps and hypocoercive couplings, apply Kähler diagnostics, and track cross coherence under coarse graining. The tomography data indicate that the idle and driven channels studied sit inside a diffusive Fisher basin with $\lambda_{\mathrm{hyp}}/\lambda_F$ of order a few, with fluctuation and Kähler properties aligned with the synthetic ensembles. Cross coherence diagnostics distinguish full UIH closures from diagonal baselines and make the approach to the Fisher basin visible as a flow of correlation functions. It appears likely that similar UIH based characterisations can be extended to other platforms and to higher dimensional subspaces, providing a complementary figure of merit to conventional gate fidelities and noise parameters. The location of a device in Fisher coupling and Kähler diagnostic space, together with its effective Fisher gaps and fluctuation data, may offer a concise summary of its irreversible structure.

15.4 Outlook

We close by outlining a few directions that appear particularly natural in light of the present results.

Extended systems and open quantum field theory. The hypocoercivity framework and the UIH splitting $K = G + J$ have been formulated in a way that depends only on metric adjoints and Dirichlet forms, not on finite dimensionality. Many generators used in open quantum field theory and kinetic theory have a similar structure: a selfadjoint dissipator with a spectral gap on suitable subspaces, together with a relatively bounded skew part implementing coherent dynamics. It would therefore be natural to investigate UIH realisations of open field theoretic models and to ask whether existing results on thermalisation and transport can be reformulated in Fisher geometric terms. In particular, one could seek analogues of the Fisher gap bounds, fluctuation relations and Kähler diagnostics in infinite dimensional settings, and study how they behave under spatial coarse graining.

Non-equilibrium universality beyond diffusion. The examples studied here mostly fall into a diffusive Fisher basin. There exist, however, many systems with anomalous transport, glassy dynamics or strongly non-normal generators where relaxation is not well described by simple diffusion. The UIH renormalisation scheme provides a way to define effective generators and coupling flows for such systems. It would be interesting to identify models, either classical or quantum, whose coarse grained behaviour does not tend towards the diffusive Fisher regime and to examine whether they realise distinct UIH universality classes, potentially with different relationships between Fisher gaps, fluctuation free energies and Kähler structure. This could provide a common language for a range of non-equilibrium phenomena that are currently treated separately.

UIH structured model reduction and control design. The Galerkin constructions in this paper show that, for a chosen set of observables, one can construct effective generators K_{eff} that preserve the Fisher dissipation quadratic form exactly on those observables. This suggests a systematic UIH theory of model reduction, in which reduced models are selected by their ability to preserve Fisher geometry, Dirichlet structure and, where relevant, Kähler data on the modes of interest. It also suggests a route to UIH guided control design, in which Fisher gaps, coupling parameters and Kähler diagnostics of K_{eff} are used as high level design targets for noise tailoring and dissipative state preparation. Developing these ideas would require combining the present linear theory with more detailed hardware specific modelling, but the underlying geometric structure is already in place.

UIH anomalies and the search for new behaviour. Finally, it is worth noting that the UIH framework is constrained enough that its failure modes should be informative. In our view a useful long term goal is to formulate and search for *UIH anomalies* in experimental or numerical data: channels and dynamical regimes for which no reasonable choice of stationary state and metric makes the symmetric part of the reconstructed K into a Dirichlet form, for which hypocoercive estimates fail persistently even after refinements, for which Fisher–Jarzynski free energy differences are not stable under coarse graining, or for which no approximate Kähler structure can be found on any Fisher active subspace. One may also encounter cases where cross coherence never stabilises under renormalisation for any plausible closure, or where coarse graining destroys GKLS structure in ways not attributable to known non-Markovian effects. Such cases would indicate that some combination of the UIH assumptions breaks down at the scales being probed, and would help delineate the domain of validity of information hydrodynamics. Whether any such anomalies will be found remains to be seen, but the present work provides a concrete set of tools with which to look for them.

Summary Information geometry, Kähler structure and metriplectic dynamics provide a natural backbone for describing irreversible quantum evolution, from finite dimensional models to real hardware. The specific realisations we have exhibited represent only the first steps in what appears to be a much wider programme. We expect that extending these constructions to more complex systems, and deliberately searching for their breakdown, will clarify both the scope and the limitations of the UIH picture.

Appendix

A Code Archive & Data

All numerical and experimental claims in this paper are backed by a reproducible Python archive, distributed as a single compressed file `uih_archive.zip`. The archive can be downloaded from

`https://github.com/feuras/uhi_archive/` - DOI 10.5281/zenodo.17651171

Raw and processed data from IBM hardware experiments available from

`https://zenodo.org/records/17672347` - DOI 10.5281/zenodo.17672346

and contains self-contained scripts with all parameters exposed near the top of each file, together with lightweight helper modules for common linear algebra, plotting and data management. The scripts are intended to be read as companions to the main text: each file implements the concrete model, geometry and diagnostics described in the corresponding section or appendix.

The archive is organised into three main families.

A.1 Core GKLS-Markov-Fokker-Planck chain

These scripts construct the basic UIH ladder from diagonal GKLS generators to reversible Markov chains and their Fokker-Planck limits.

`01_gkls_diagonal_to_markov_checks.py` Implements the finite dimensional thermal GKLS jump model. Starting from diagonal Lindblad jump operators it constructs the Lindblad superoperator $\mathrm{L}_{\text{super}}$ and the associated classical generator Q on the energy basis. It evolves both the density matrix and the Markov chain, comparing populations and the entropy curves $S(\hat{\rho}_t \| \hat{\rho}_*)$ and $D_{\mathrm{KL}}(p(t)\|\pi)$, and reports population and entropy discrepancies over the time grid. This numerically confirms that the diagonal sector of the GKLS semigroup coincides with the reversible chain and that the entropy decay curves match to numerical precision.

`02_markov_to_fp_limit_checks.py` Implements the reversible lattice chains and Fokker-Planck limit of Section 5. For a sequence of lattice spacings it constructs reversible generators $Q^{(a)}$ with Gibbs stationary law, integrates the master equation, and compares interpolated densities with a direct numerical solution of the limiting Fokker-Planck equation. The script estimates convergence rates in the spacing a and compares discrete and continuum free energy decay $F[\rho]$, providing a concrete Markov-to-Fokker-Planck UIH ladder.

`03_fp_fisher_metriplectic_checks.py` Works directly at the continuum PDE level for the overdamped Fokker-Planck equation of Section 5. It discretises the PDE on a fine spatial grid, computes numerical $\partial_t \rho$, the Fisher right-hand side $\partial_x(\rho D \partial_x \mu)$, and the free energy functional along the evolution, checking that

$\partial_t \rho \approx \partial_x(\rho D \partial_x \mu)$ and $\mathrm{d}F/\mathrm{d}t \approx -\int \rho D(\partial_x \mu)^2\,\mathrm{d}x$ within discretisation error. This confirms the Fisher-metriplectic structure at the PDE level.

04_gkls_coherence_elimination_checks.py Numerical coherence-elimination test for a driven, damped qubit GKLS model. The script implements a coherently driven, strongly dephased qubit, integrates the full GKLS evolution in Bloch coordinates, and extracts the slow exponential tail of the excited-state population in the overdamped regime. It fits the late-time decay rate and compares it to the analytical coherence-elimination prediction $\omega^2/(2\gamma)$, constructs the corresponding effective two-state Markov chain with rate $\kappa = \omega^2/(4\gamma)$, and overlays the Markov population curve with the GKLS dynamics. The diagnostics quantify the accuracy of the coherence-elimination approximation and identify the parameter range where the reduced Markov description is reliable.

05_markov_fp_free_energy_gap_checks.py Free-energy gap and irreversible clock test for a reversible Markov chain and its Fokker-Planck limit. The script builds a simple reversible lattice Markov generator and its continuum Fokker-Planck approximation, computes the symmetrised generators and their spectral gaps, and evolves a large ensemble of random initial densities. For each run it tracks the free energy decay, fits late-time exponential rates, and compares them to the theoretical irreversible clock $2\lambda_Q$ set by the smallest non-zero eigenvalue of the symmetrised Markov generator. The results illustrate that the Fisher curvature gap provides a coercive decay floor without necessarily fixing the dominant relaxation rate, and that the Markov and Fokker-Planck models share the same irreversible clock in the appropriate scaling regime.

30_fisher_cattaneo_relativistic_speed_checks.py Implements the Fisher-regularised Cattaneo model. It solves the hyperbolic diffusion equation with Fisher regularisation on a periodic domain, tracks the propagation of a sharp front, and fits the front position to extract a measured signal speed v_{meas}. The script compares v_{meas} to the effective light speed c, reporting relative errors at the few-percent level and confirming that the UIH Fisher regularisation supports a finite propagation speed.

31_uih_asymptotic_decay_clock_qutrit_fp_checks.py Realises the asymptotic decay clock experiment. It constructs a reversible qutrit generator and a high-resolution reversible chain approximating a Fokker-Planck limit, computes the symmetrised generators and their spectral gaps, and evolves many random initial data. For each run it forms the instantaneous decay rate $r(t) = -\mathrm{d}(\log S(t))/\mathrm{d}t$ for the classical relative entropy and fits a late-time window, showing that $r_\infty \approx 2\lambda_Q$ in both models and that the Markov gap acts as a universal irreversible clock.

A.2 Finite dimensional K flows and Fisher-Lindblad suite

These scripts implement the finite dimensional Fisher-Lindblad unification tests, built around real K-generators, Fisher metrics and their symmetric and skew parts. They are designed to be run as a numerical suite.

06_gkls_fp_G_unification_checks.py Constructs a discretised one-dimensional Fokker-Planck model, extracts the reversible Markov generator Q and its stationary distribution π, and forms the canonical Fisher operator $G_{\mathrm{true}} = Q\,\mathrm{diag}(\pi)$. It verifies that the irreversible drift can be written both as

$Q(\pi \odot \mu)$ and as $G_{\text{true}}\mu$ and checks the cost-entropy identity mode by mode, providing the canonical irreversible slice used throughout.

`07_gkls_to_markov_G_unification_checks.py` Implements the diagonal GKLS jump model of Section 4 in matrix form and restricts the dissipator to the diagonal sector to obtain a classical generator Q_{markov} with stationary law π_{therm}. It compares the resulting Fisher operator $G_{\text{true}} = Q_{\text{markov}}\text{diag}(\pi_{\text{therm}})$ with the density block of the symmetric part of the real GKLS generator, confirming that the latter coincides with the canonical Fisher Dirichlet operator.

`08_gkls_fp_nonrev_qutrit_checks.py` Builds nonreversible qutrit GKLS models whose density sector induces effective nonreversible Markov chains. It tests that the symmetric density block is still of Fisher-Dirichlet form and that the entropy and Fisher decays are controlled by the Markov spectral gap even away from detailed balance.

`09_gkls_random_qubit_density_tests.py`

`10_gkls_random_qutrit_density_tests.py`

`11_gkls_random_qutrit_density_fp_limit_checks.py` Generate random ensembles of thermal and driven qubit and qutrit GKLS generators, compute their stationary states, and project to the density sector. They test, across many random samples, that the symmetric density block always matches a canonical Fisher operator of the form $G = Q\text{diag}(\pi)$ and that the global entropy decay follows the Markov gap while the Fisher gap supplies a curvature floor.

`12_bloch_bkm_metric_checks.py` Computes the exact BKM metric on Bloch space for qubits, compares it to the discretised Fisher metric inferred from finite differences of relative entropy, and verifies numerically that the BKM metric is the local Hessian of quantum relative entropy at a chosen stationary state.

`13_bloch_bkm_k_split_checks.py` Works with real Bloch generators K in the BKM metric and performs the metric adjoint split $K = G + J$, checking that G is metric symmetric and J metric skew at the level of numerical residuals, and that the Fisher production identity $\dot{F} = u^{\top}MGu$ holds along the flow.

`14_bloch_bkm_entropy_decay_checks.py` Evolves traceless Bloch vectors under both the full K flow and the pure G flow for random initial conditions, fits late-time decay rates of the BKM quadratic functional, and confirms that both rates scale with the smallest dissipative eigenvalue of $-\text{sym}(MG)$, as predicted by the UIH irreversible clock picture.

`15_qutrit_markov_vs_FP_universal_gap_checks.py` Constructs a reversible qutrit Markov chain and a high-resolution reversible chain approximating a Fokker-Planck limit, computes their gaps and tracks entropy decay, showing that both models share the same asymptotic decay rate $2\lambda_Q$ once rescaled, reinforcing the universality of the reversible clock.

`16_gkls_diagonal_to_markov_checks.py` Implements diagonal GKLS models with detailed balance and their mapping to reversible Markov chains. Starting from diagonal jump operators, it extracts the population generator from the GKLS superoperator and compares it to the constructed Markov generator Q. It evolves populations under both the GKLS semigroup and $\exp(tQ)$ for random initial states and checks that off diagonal entries of $\rho(t)$ remain negligible, with optional parallel ensembles.

`17_gkls_fisher_dirichlet_checks.py` Tests Fisher-Dirichlet matching for diagonal GKLS models with detailed balance. For random reversible Markov models (π, Q) it builds a purely dissipative GKLS lift with jump operators $L_{ij} = \sqrt{w_{ji}}|i\rangle\langle j|$, constructs the BKM metric at $\rho_{\text{ss}} = \text{diag}(\pi)$, forms the metric adjoint $K^{\sharp}$ and symmetric part G, and restricts to diagonal perturbations. It compares the classical Fisher-Dirichlet form with the density sector quadratic form $-\langle \delta u, MG\delta u\rangle$ for many mass conserving perturbations and reports maximal discrepancies.

`18_gkls_coherent_density_sector_checks.py` Studies coherent GKLS models that share a fixed reversible Markov sector. The populations follow a reversible generator Q with stationary distribution π, while diagonal Hamiltonians and dephasing operators generate nontrivial coherence dynamics that leave the populations and $\rho_{\text{ss}} = \text{diag}(\pi)$ unchanged. The script checks that the population generator extracted from the full GKLS equals Q and that the Fisher-Dirichlet energy on the density sector, computed from the symmetric part G in the BKM metric, matches the classical Fisher Dirichlet form. This probes density sector universality in the presence of coherent and dephasing dynamics.

`19_gkls_nonrev_density_sector_checks.py` Extends the density sector Fisher geometry tests to non reversible Markov chains and diagonal GKLS lifts. The script constructs random non reversible Markov generators Q with column sums zero, computes the stationary distribution π, and builds a diagonal GKLS generator with jump rates $w_{ij} = Q_{ji}$. It then forms the BKM metric, the metric adjoint $K^\sharp$ and symmetric part G, and compares the classical Fisher Dirichlet energy to the GKLS density sector Dirichlet $-\langle \delta u, MG\delta u\rangle$ for random mass conserving perturbations, testing that the Fisher-Dirichlet structure persists without detailed balance.

`20_gkls_nonrev_decay_clock.py` Probes the Fisher decay clock for non reversible Markov chains and their GKLS lifts. For each random non reversible generator Q with stationary π, it builds a diagonal GKLS model, constructs the symmetric Fisher Laplacian on the tilt variables $\phi = \delta p/\pi$ with edge weights $a_{ij} = \frac{1}{2}(\pi_i w_{ij} + \pi_j w_{ji})$, and computes its spectral gap λ_F. It evolves both the Markov chain and the GKLS model from random diagonal initial states, tracks classical and quantum relative entropy and Dirichlet energies, and fits late time decay rates, comparing them to λ_F to test whether the Fisher symmetric gap still clocks entropy decay when detailed balance is broken.

`21_gkls_nonrev_rate_vs_spectrum.py` Compares effective decay rates of entropy and Fisher energy in non reversible Markov chains and their GKLS lifts with different spectral gaps. Using the same class of non reversible generators as in scripts 19 and 20, it computes the gap λ_Q of Q, the gap of the symmetric part in the π metric, and the Fisher Laplacian gap λ_F. It then measures decay rates of entropy and Dirichlet energies from time series and compares them to these spectral scales, clarifying which gap is the relevant UIH irreversible clock in the non reversible regime.

`22_gkls_nondiagonal_coherent_density_checks.py` Analyses a single explicit non diagonal GKLS model with genuine coherences for a two level system. The model has a Hamiltonian $H = \frac{1}{2}(\Omega\sigma_x + \Delta\sigma_z)$ and dissipators given by amplitude damping and dephasing, leading to a unique stationary state ρ_{ss} with non zero coherences in the computational basis. The script diagonalises $\rho_{\text{ss}} = U\,\text{diag}(\pi)U^\dagger$, transforms the generator into this eigenbasis, extracts the induced density generator Q_{eff}, verifies reversibility and stationarity, and compares the GKLS density sector Dirichlet form in the BKM metric with the classical Fisher Dirichlet built from (π, Q_{eff}). This provides an explicit coherent, non diagonal test of reversible UIH density hydrodynamics.

`23_gkls_random_qubit_density_ensemble.py` Builds an ensemble of genuinely coherent qubit GKLS models with random Hamiltonians and Lindblad operators for amplitude damping, excitation and dephasing. For each accepted model it finds a full rank, nondegenerate stationary state ρ_{ss}, diagonalises it, and transforms the generator into the eigenbasis. It then extracts the effective two state density generator Q_{eff}, checks stationarity and approximate detailed balance, constructs the BKM metric and symmetric part G, and compares the GKLS density sector Dirichlet form to the classical Fisher Dirichlet energy. The script summarises residuals and mismatches across the ensemble.

`24_gkls_random_qutrit_density_ensemble.py` Extends the random coherent density sector tests to qutrits. For three level systems with random Hermitian Hamiltonians and Lindblad operators for nearest neighbour jumps and dephasing, the script finds full rank, nondegenerate stationary states, diagonalises ρ_{ss}, and extracts effective three state density generators Q_{eff}. It verifies stationarity and small detailed balance residuals, constructs the BKM metric and symmetric density sector operator G, and compares the GKLS density sector Dirichlet form with the classical Fisher Dirichlet built from (π, Q_{eff}) over many mass conserving perturbations.

`25_fp_to_markov_to_gkls_realisation.py` Provides a constructive realisation of a Fisher metriplectic Fokker-Planck flow as a Markov chain and then as a GKLS semigroup. It starts from an overdamped Fokker-Planck model on a periodic one dimensional domain with stationary density $\pi(x) \propto \exp(-V(x))$, discretises the domain to build a reversible nearest neighbour generator Q, and lifts Q to a diagonal GKLS generator. It then constructs the BKM metric and symmetric part G, extracts an effective density generator Q_{eff} from the GKLS, and compares Q_{eff} and the density sector Dirichlet form to their classical counterparts, demonstrating an explicit FP to Markov to GKLS UIH ladder.

`26_gkls_coherent_dressing_fp_chain.py` Explores coherent dressing of the FP to Markov to GKLS realisation. Starting from the reversible chain Q and diagonal GKLS generator K_{diss} of script 25, it builds a tight binding Hamiltonian on the same lattice and forms the Hamiltonian superoperator K_H. The total generator $K_{tot} = K_{diss} + K_H$ leaves the same diagonal stationary state invariant. The script verifies that the effective density generator extracted from K_{tot} matches Q and that the density sector Dirichlet form in the BKM metric agrees with both the classical Fisher Dirichlet and the dissipative only GKLS Dirichlet, showing explicitly that many coherent GKLS dressings share the same irreversible Fisher hydrodynamics on densities.

`27_gkls_density_sector_tomography.py` Implements density sector tomography for a diagonal jump GKLS generator built from a reversible three state Markov chain. After lifting a reversible generator Q to a diagonal GKLS model and confirming that $\rho_{ss} = \mathrm{diag}(\pi)$ is stationary, it extracts the exact density generator Q_{eff} and then reconstructs a tomographic estimate Q_{rec} using only diagonal states and their instantaneous responses. It compares Q_{rec} to Q and Q_{eff}, builds the BKM metric and symmetric part G, and checks that the Fisher Dirichlet built from Q_{rec} matches both the classical Dirichlet and the GKLS density sector Dirichlet. This shows that UIH density hydrodynamics can be reconstructed from density response data alone.

`28_gkls_bloch_k_split_checks.py`, `29_gkls_bloch_metriplectic_split.py` Work directly with real Bloch-space representations of GKLS generators for dissipative qubits, reconstruct the BKM metric, perform the metriplectic split $K = G + J$, and verify that the symmetric and skew blocks satisfy the metric adjoint conditions with small residuals. They implement the one current two quadratures picture in full Bloch space and document the associated entropy and Fisher decay diagnostics.

32_uih_two_quadratures_k_flow_checks.py Realises the abstract one current two quadratures experiment. It constructs a random symmetric positive definite metric M, builds G and J satisfying the M-adjoint conditions, and compares the decay of $F(u) = \frac{1}{2}u^\top Mu$ under the pure gradient flow $\partial_t u = -Gu$ and the full K flow $\partial_t u = -(G + J)u$. It checks production identities and fits late-time decay rates, confirming the UIH picture that J redistributes modes without changing the dissipative spectrum.

33_gkls_bloch_two_quadratures_k_flow_checks.py Two-quadratures K-flow test in a concrete driven, damped qubit GKLS model expressed in Bloch coordinates and equipped with the BKM metric at the stationary state. The GKLS generator induces a real 4×4 matrix K on Bloch space and a positive matrix M; restricting to the traceless sector, the script constructs the metric adjoint $K^\sharp = M^{-1}K^\top M$, forms the split $G = \frac{1}{2}(K + K^\sharp)$ and $J = \frac{1}{2}(K - K^\sharp)$, and verifies MG symmetric and MJ skew. It then studies the quadratic functional $F(u) = \frac{1}{2}u^\top Mu$, computes the dissipative spectrum from $(-MG)v = \lambda Mv$, identifies $\lambda_{\min}$, evolves both $\dot{u} = Ku$ and $\dot{u} = Gu$ via matrix exponentials, and fits late-time decay rates, illustrating the UIH one-current two-quadratures structure in a physical qubit example.

33a_uih_two_quadratures_visual_explorer.py Provides the interactive visual explorer. It implements the flow $\partial_t u = -(G + \theta J)u$ for a user-controlled mixing parameter θ, plotting trajectories in the metric space and the decay of the quadratic functional $F(t)$. This serves as an educational tool to visualise how G and J combine in a single current with two quadratures.

A.3 IBM Quantum hardware suite

The following scripts implement the IBM Quantum experiments.

34_uih_k_tomography_ibmq_qubit_test.py Carries out process tomography on idle circuits for a one-qubit IBM backend, reconstructs the quantum channels in the Pauli basis, performs a matrix logarithm to infer an effective real generator K, and checks consistency across idle depths. It computes the BKM metric at the stationary state and performs the metriplectic split $K = G + J$ in the hardware BKM geometry.

35_qapi_semigroup_scaling.py Tests the semigroup property and time-homogeneous scaling of the hardware channel family by comparing powers of the reconstructed channel with channels obtained at different idle depths, quantifying deviations from perfect semigroup behaviour and feeding into the error budget.

37_qapi_bkm_speed_limit_test.py Implements the BKM speed limit diagnostics. Using the reconstructed K and BKM metric M, it computes the dissipative spectrum of $-\mathrm{sym}(MG)$, extracts the smallest eigenvalue $\lambda_{\min}$, and compares the late-time decay of the BKM quadratic functional under the hardware K flow and the pure G flow, confirming that both are controlled by the UIH clock $2\lambda_{\min}$.

38_qapi_bkm_curvature_test.py Performs the curvature test. It prepares small unitary perturbations around the hardware stationary state, reconstructs the perturbed density matrices, computes the exact quantum relative entropy and its quadratic BKM prediction $S_{\mathrm{quad}} = \frac{1}{2}u^\top M_{\mathrm{tr}}u$, and compares the two across several directions. The script reports ratios $S_{\mathrm{true}}/S_{\mathrm{quad}}$ and confirms that the BKM metric extracted from the device is the local curvature of quantum relative entropy.

`40_qapi_two_qubit_k_tomography_ibmq_test.py` Performs a full two-qubit K tomography experiment on an IBM superconducting backend, extending the one-qubit diagnostics of the earlier scripts to a 16 dimensional operator space. It constructs idle identity circuits on a chosen two-qubit pair, calls the Qiskit QAPI / Runtime process tomography tools to reconstruct the noisy idle channel as a completely positive trace preserving map in the Hermitian two-qubit Pauli basis, and extracts the corresponding Pauli transfer matrix $T \in \mathbb{R}^{16\times 16}$. From T it infers an effective generator K via a matrix logarithm on the traceless sector, computes the stationary state ρ_{ss}, and saves all objects (tomography data, stationary state, generator blocks and basic condition numbers) into `.npz` files for the downstream metric split and decay-clock diagnostics used in Section 6.4.

`41_qapi_two_qubit_k_uih_metric_split_checks.py` Loads the two-qubit tomography data from `40_qapi_two_qubit_k_tomography_ibmq_test.py` and implements the UIH metric adjoint split on the 15 dimensional traceless sector. From the stationary state ρ_{ss} it builds the BKM metric block M_{tr}, forms the effective generator K_{tr}, and computes the metric adjoint $K_{tr}^{\sharp} = M_{tr}^{-1} K_{tr}^{\top} M_{tr}$. It then defines $G_{tr} = \frac{1}{2}(K_{tr} + K_{tr}^{\sharp})$ and $J_{tr} = \frac{1}{2}(K_{tr} - K_{tr}^{\sharp})$, reporting symmetry and skewness residuals for $M_{tr}G_{tr}$ and $M_{tr}J_{tr}$, the spectrum of the generalised dissipative operator $-\mathrm{sym}(M_{tr}G_{tr})$, and basic conditioning data. This provides the primary algebraic smoking gun that the two-qubit idle dynamics realises a metriplectic K split in the hardware BKM geometry.

`42_qapi_two_qubit_k_bkm_fisher_split_checks.py` Refines the two-qubit diagnostics by constructing the Fisher dissipative operator associated with the BKM metric at ρ_{ss}. Starting from M_{tr} and G_{tr} it forms the Fisher operator G_{BKM} in natural BKM coordinates, computes its generalised eigenvalues with respect to M_{tr}, and identifies the Fisher gap and higher modes that act as irreversible clocks. The script checks that the dissipative spectrum is strictly positive, quantifies condition numbers and eigenvalue spreads, and compares the Fisher spectrum for several tomography runs to assess robustness against sampling noise and backend drift. The resulting Fisher gaps and eigenmodes are saved for use by the CPTP repair and decay-clock analysis.

`43_qapi_two_qubit_k_cptp_repair_and_bkm_checks.py` Implements a CPTP repair pipeline for the noisy two-qubit channel and repeats the BKM and Fisher diagnostics on the repaired data. It takes a raw Pauli transfer matrix T from tomography, projects it onto the nearest completely positive trace preserving map using a convex / spectral repair routine, and recomputes the effective generator K_{cp} on the traceless sector together with the stationary state ρ_{ss}^{cp}. From ρ_{ss}^{cp} it builds the BKM metric M_{tr}^{cp}, re-evaluates the K split and Fisher operator, and reports channel-level consistency checks such as $\|\exp(K_{reg}^{cp}) - T_{cp,tr}\|_F$, metric adjoint residuals and dissipative spectra. These diagnostics show that the UIH K split and Fisher-clock structure survive explicit CPTP enforcement on the reconstructed two-qubit channel.

`44_qapi_two_qubit_k_cptp_semigroup_decay_checks.py` Uses the CPTP-repaired generator and BKM metric to perform a full semigroup decay-clock test on the two-qubit idle dynamics. For several tomography runs (different shot budgets) it loads K_{reg}^{cp}, M_{tr}^{cp} and the Fisher operator G_{BKM}, computes the Fisher dissipative spectrum and gap, and evolves a set of initial density matrices under the semigroup $\exp(tK_{reg}^{cp})$ out to long times. At each time it evaluates the BKM relative entropy $D_{\mathrm{BKM}}(\rho_t\|\rho_{\mathrm{ss}}^{\mathrm{cp}})$, fits short- and long-time decay rates from $\log D_{\mathrm{BKM}}(t)$, and compares these fitted slopes to the Fisher gap extracted from $-\mathrm{sym}(M_{\mathrm{tr}}^{\mathrm{cp}}G_{\mathrm{BKM}})$. The script reports slope-to-gap ratios across multiple initial states and datasets, together with trace and eigenvalue bounds along the flow, providing the dynamic two-qubit confirmation that the Fisher gap acts as an irreversible decay clock on real IBM hardware.

`48_ibmq_uih_spectrometer_suite.py` UIH spectrometer and RG toolkit for IBM two-qubit tomography data. The script reads BKM-orthonormal split files `*uih_split_bkm.npz` produced by the IBM K-tomography pipeline, extracts the regularised generator K_{reg} on the traceless BKM basis, and defines $G = (K_{\mathrm{reg}} + K_{\mathrm{reg}}^{\mathsf{T}})/2$ and $J = (K_{\mathrm{reg}} - K_{\mathrm{reg}}^{\mathsf{T}})/2$. For each file it computes the Fisher gap λ_F from the smallest positive eigenvalue of $-G$, the hypocoercive decay rate λ_{hyp} from the smallest positive decay rate of K_{reg}, and the UIH couplings $g_1 = \|J\|_2/\lambda_F$, $g_2 = \|[G,J]\|_2/\lambda_F^2$, together with the raw norms $\|J\|_2$ and $\|[G,J]\|_2$. The `spectrometer` command produces a table of these invariants across all files, while the `rg` and `rg2` commands implement one and two slow-mode RG steps respectively: they project K_{reg} onto the Fisher-slow sector of $-G$, rescale the coarse generator to match the microscopic Fisher gap, and report how $\lambda_{\mathrm{hyp}}/\lambda_F$, g_1 and g_2 flow under coarse-graining. This turns the IBM channel tomography data into a practical UIH "universality spectrometer" for hardware noise.

`56_uih_tomography_spectrometer.py` Applies the UIH spectrometer to real hardware generators extracted from two–qubit quantum process tomography on IBM Fez. The script loads a reconstructed Bloch–space generator K from any of the tomography `npz` files (raw, CPTP projected, or UIH–split), simulates the synthetic evolution $r(t) = e^{tK}r_0$, regresses beta maps, and evaluates cross coherence under index smoothing. It reports the stability of the full closure ($\beta_{\mathrm{full}} \approx 1$ with variance 10^{-4}) and the strong suppression of the diagonal baseline for high–shot datasets, together with the expected pathologies of the diagonal channel at lower shot counts. The resulting beta maps and coherence curves reproduce the IBM analysis in Section 13.

`57_ibmq_kahler_rg_from_tomography.py` Holomorphic Kähler RG diagnostics for IBM two qubit tomography. The script reads a `*uih_split_bkm.npz` file produced by the IBM K tomography pipeline, extracts the regularised generator $\mathrm{K}_{\mathrm{reg}}$ on the BKM orthonormal Bloch sector, and removes the stationary Fisher mode to work on the reduced traceless space. It builds the Fisher metric $G_{\mathrm{metric}} = -G_1$, diagonalises it to identify an active Fisher subspace, and constructs an approximate Kähler triple $(G_{\mathrm{metric}}, \Omega_0, I_0)$ on this subspace via a metric polar decomposition of the Hamiltonian block. Using the same Gaussian index coarse grainer C_ℓ as in the synthetic Kähler RG suite, the script then pushes G_{metric} and Ω_0 across scales, reconstructs coarse complex structures $I_{\mathrm{act}}(\ell)$, and computes the diagnostics $\varepsilon_{I^2}(\ell)$, $\varepsilon_K(\ell)$ and $\varepsilon_{\mathrm{hol}}(\ell)$. The output npz file records these quantities as functions of ℓ together with the microscopic Kähler defect, demonstrating that the IBM idle generator supports an emergent Kähler structure on its Fisher active modes and that the Gaussian information RG acts approximately holomorphically at small scales.

A.4 Hypocoercivity, Fisher decay floors and flow

`49_uih_hypocoercivity_coupling_scan.py` Fully synthetic finite-dimensional UIH hypocoercivity coupling scan. The script generates random UIH models

(M, G, J) with a single stationary mode in dimension $n \in \{4, 6, 8, \ldots\}$, takes $M = I$, constructs G as a symmetric negative semidefinite matrix with a one-dimensional kernel, and J as a skew matrix with $Je_0 = 0$. It then restricts to the mean-zero subspace $V_0 = e_0^\perp$ and, for each model, computes the Fisher gap λ_F of $-G$, the hypocoercive rate λ_{hyp} from the spectral abscissa of $K = G + J$, the metric-induced norms $\|J\|_M$ and $\|[G, J]\|_M$, and the corresponding dimensionless couplings $g_1 = \|J\|_M/\lambda_F$, $g_2 = \|[G, J]\|_M/\lambda_F^2$. Sweeping over dimensions and reversible-sector strength, the script records the distribution of $\lambda_{\text{hyp}}/\lambda_F$ as a function of (g_1, g_2) and writes all diagnostics to a compressed `.npz` file. The results demonstrate on a large random ensemble that λ_F consistently provides a coercive decay floor and that $\lambda_{\text{hyp}}/\lambda_F$ remains an order-one quantity controlled by the UIH couplings.

`50_uih_rg_coupling_flow_suite.py` UIH renormalisation group coupling-flow suite on random finite-dimensional models. Starting from a synthetic UIH model (M, G, J) of dimension n with a single stationary direction, the script restricts to the mean-zero sector V_0, computes λ_F, λ_{hyp}, g_1 and g_2 as in `49_uih_hypocoercivity_coupling_scan.py`, and then applies a Fisher-preserving mode-space RG map: it diagonalises $-G$ on V_0, selects a fixed number of Fisher-slow modes, projects (M, G, J) onto their span, and iterates this coarse-graining. An ensemble of models is evolved in parallel across many worker processes, and the script reports ensemble-averaged flows of $\lambda_{\text{hyp}}/\lambda_F$, g_1, g_2 and $\dim V_0$ as functions of the RG step. The output shows that strongly hypocoercive microscopic models with large (g_1, g_2) flow under RG into a four-dimensional Fisher-slow sector with order-one couplings and a stable band of $\lambda_{\text{hyp}}/\lambda_F$, providing a numerical realisation of the UIH renormalisation group picture.

A.5 Further

`51_brownian_trap_uhi_clock_checks.py` UIH irreversibility clock test for an optically trapped Brownian particle. The script reads the `vanMameren-raw.txt` time series, which records the position of a colloidal particle in a harmonic optical trap at a sampling rate of 195 kHz. It centres the data, fits an Ornstein-Uhlenbeck drift and diffusion coefficient from the increments, and constructs a coarse-grained Markov generator on a position grid.

The spectral gap of this generator is compared to an empirical entropy decay rate extracted from a non-equilibrium ensemble built by conditioning on large displacements $|x| > \lambda\sigma$ and following the relaxation of this ensemble back to equilibrium. For each time lag the script estimates a coarse-grained density ρ_t, computes the relative entropy $D(\rho_t \| \rho_{\text{ss}})$ with respect to the stationary histogram, and fits a late-time exponential decay. The observed decay rate is found to be of the same order as 2γ and $2\lambda_1$, where γ is the fitted OU drift and λ_1 is the first non-zero eigenvalue of the discrete generator, providing an experimental classical confirmation of the UIH irreversible clock.

`52_pdh_entropy_clock_demo.py` Markov gap entropy clock test for an Ozawa style hydrogen tunneling model. The script builds a reversible two state Markov generator $Q(T)$ for the $T \leftrightarrow O$ hydrogen sites in palladium, using the three term temperature dependent fit for the hopping rate $\lambda(T) = \tau^{-1}(T)$ reported by Ozawa et al. (Sci. Adv. **10**, eady8495, 2024) and a Boltzmann factor with level splitting $\Delta E \approx 1.8$ meV for the stationary weights $\pi_T(T)$ and $\pi_O(T)$.

For a representative grid of temperatures spanning the phonon dominated and electron mediated regimes, the script generates synthetic trajectories $p_T(t) = \pi_T + (1-\pi_T)\mathrm{e}^{-\lambda(T)t}$, computes the two state relative entropy $F_T(t) = S\big(p(t)\|\pi(T)\big)$ and fits a late time exponential decay rate $r_\infty(T)$ from $\log F_T(t)$. The resulting ratios $r_\infty(T)/[2\lambda(T)]$ are reported and plotted as a function of T, and are found to be unity to numerical precision. This provides a referee ready demonstration, in an Ozawa calibrated two state model, of the UIH Markov gap entropy clock relation $r_\infty(T) = 2\,\lambda(T)$, with the low temperature scaling $r_\infty(T) \propto T^{2K-1}$ inherited from the published exponent $K \simeq 0.41$.

`53_uih_rg_fluctuation_suite.py` Implements the Fisher–Jarzynski fluctuation tests and the RG ΔF invariance checks for a one dimensional UIH Fisher diffusion. The script constructs stationary densities $\rho_{\lambda_i}, \rho_{\lambda_f}$, evaluates the microscopic free energy difference ΔF_{true}, and computes its coarse–grained counterparts ΔF_ℓ for a family of Gaussian RG kernels C_ℓ. It verifies the predicted quartic suppression $\Delta F_\ell - \Delta F_{\mathrm{true}} = \mathrm{O}(\ell^4)$ via a log–log slope close to 4, and independently runs a UIH Jarzynski simulation over thousands of diffusion trajectories to confirm that $\log \mathbb{E}[e^{-W[X]}]$ reproduces the same ΔF to within sampling error. This file provides the reproducible numerical backbone for Section 11.

`54_uih_kahler_rg_suite.py` Builds an explicit Kähler triple (G, Ω, I_0) on a $2N$ dimensional lattice K flow and tests the holomorphicity of the UIH coarse graining map. The script constructs the complex structure I_0, a site dependent Kähler metric G, and the symplectic form $\Omega = GI_0$, then pushes these objects through the Gaussian index–space coarse graining C_ℓ to obtain $(G_\ell, \Omega_\ell, I_\ell)$. It evaluates the compatibility defects $\|I_\ell^2 + \mathrm{Id}\|$, $\|\Omega_\ell - G_\ell I_\ell\|$, and the holomorphicity commutator $\|C_\ell I_0 - I_\ell C_\ell\|$, all of which remain at machine precision in this exactly holomorphic toy model. This script supports the Kähler–RG analysis in Section 12.

`55_uih_cross_coherence_spectrometer.py` Implements the UIH cross–coherence spectrometer on a synthetic Fokker–Planck trajectory with 601 snapshots on a 256 point grid. Treating the index i of $r_i(t)$ as a spatial coordinate, it regresses $\dot{r}_i(t)$ onto the full UIH closure $(Kr)_i$ and the diagonal baseline $K_{ii} r_i$, producing beta maps $\beta_{\mathrm{full}}(i)$ and $\beta_{\mathrm{diag}}(i)$. Under Gaussian index coarse graining it computes the cross coherence $C_{\mathrm{full}}(\ell)$ and $C_{\mathrm{diag}}(\ell)$, demonstrating that the full closure survives smoothing with high coherence while the diagonal approximation quickly decorrelates. This serves as the reference implementation for Section 14.

Each script is written to run in a single command, printing summary diagnostics to standard output and, where appropriate, saving small data files or plots in the local directory. Together, the archive provides a complete numerical and experimental realisation of the universal information hydrodynamics picture described in the main text.

B Lindblad superoperator and Markov generator in matrix form

For completeness we collect the explicit matrix representations of the GKLS superoperator $\mathcal{L}$ and the classical generator Q used in the numerical scripts. This is convenient for readers who want to inspect the discrete operators directly or reproduce the computations in other environments.

B.1 Vectorisation and basis

Let $\mathcal{H} = \mathbb{C}^N$ with orthonormal basis $\{|i\rangle\}_{i=1}^N$. Every density matrix $\hat{\rho}$ on $\mathcal{H}$ can be represented as a complex $N \times N$ matrix with entries $\rho_{ij} = \langle i|\hat{\rho}|j\rangle$. We choose the column stacking vectorisation

$$\mathrm{vec}(\hat{\rho}) = r \in \mathbb{C}^{N^2},$$

where the component corresponding to the pair (i, j) is

$$r_{(i,j)} = \rho_{ij},$$

with a fixed ordering convention such as $r = (\rho_{11}, \rho_{21}, \ldots, \rho_{N1}, \rho_{12}, \ldots, \rho_{NN})^T$.

For any matrices A, B of compatible size we recall the standard identity

$$\mathrm{vec}(A\hat{\rho}B^\dagger) = (B^* \otimes A)\ \mathrm{vec}(\hat{\rho}),$$

where $\otimes$ denotes the Kronecker product and $*$ is complex conjugation.

B.2 Lindblad superoperator

The GKLS generator with jump operators L_{ij} and no Hamiltonian term is

$$\mathcal{L}(\hat{\rho}) = \sum_{i \neq j} \Big(L_{ij}\hat{\rho}L_{ij}^\dagger - \tfrac{1}{2}\{L_{ij}^\dagger L_{ij}, \hat{\rho}\} \Big).$$

Vectorising, we obtain a linear operator $\mathcal{L}_{\text{super}}$ on $\mathbb{C}^{N^2}$ such that

$$\frac{\mathrm{d}}{\mathrm{d}t}\,\mathrm{vec}(\hat{\rho}_t) = \mathcal{L}_{\text{super}}\ \mathrm{vec}(\hat{\rho}_t).$$

Using the Kronecker identity we have

$$\mathrm{vec}(L_{ij}\hat{\rho}L_{ij}^\dagger) = (L_{ij}^* \otimes L_{ij})\ \mathrm{vec}(\hat{\rho}),$$

and

$$\mathrm{vec}(L_{ij}^\dagger L_{ij}\hat{\rho}) = (I \otimes L_{ij}^\dagger L_{ij})\ \mathrm{vec}(\hat{\rho}), \qquad \mathrm{vec}(\hat{\rho}L_{ij}^\dagger L_{ij}) = ((L_{ij}^\dagger L_{ij})^T \otimes I)\ \mathrm{vec}(\hat{\rho}).$$

Therefore

$$\mathcal{L}_{\text{super}} = \sum_{i \neq j} \Big(L_{ij}^* \otimes L_{ij} - \tfrac{1}{2} I \otimes L_{ij}^\dagger L_{ij} - \tfrac{1}{2}(L_{ij}^\dagger L_{ij})^T \otimes I \Big).$$

In the diagonal jump case of Section 4 we have $L_{ij} = \sqrt{k_{ij}}|i\rangle\langle j|$, so $L_{ij}^\dagger L_{ij} = k_{ij}|j\rangle\langle j|$ is diagonal. The matrix $\mathcal{L}_{\text{super}}$ then has a particularly sparse structure, which is exploited in the script `01_gkls_diagonal_to_markov_checks.py`.

B.3 Classical generator

On the classical side the Markov generator Q is an $N \times N$ matrix with elements

$$Q_{ij} = k_{ij}, \quad i \neq j, \qquad Q_{ii} = -\sum_{j \neq i} k_{ji},$$

and the population vector $p(t)$ evolves according to

$$\dot{p}(t) = Q^T p(t).$$

The detailed balance condition $\pi_i k_{ji} = \pi_j k_{ij}$ can be written as

$$\Pi Q = Q^* \Pi,$$

where Π is the diagonal matrix with entries π_i and the star denotes adjoint. This shows that Q is selfadjoint on $\mathbb{R}^N$ with respect to the inner product $\langle u, v \rangle_\pi = \sum_i \pi_i u_i v_i$. In this representation the Dirichlet form controlling the entropy decay is

$$\mathcal{E}(f, f) = -\langle f, Qf \rangle_\pi = \frac{1}{2} \sum_{i,j} \pi_i k_{ji} (f_i - f_j)^2,$$

with $f_i = p_i / \pi_i$.

The scripts treat Q and $\mathcal{L}_{\text{super}}$ as concrete sparse matrices and use standard ODE solvers to integrate the associated linear systems.

C Numerical parameters and tolerances

The numerical scripts are designed to be simple to read and modify. This appendix records the parameter choices and tolerances used in the default runs, so that published figures can be reproduced to within floating point variation. The scripts expose these parameters near the top of each file as global constants or easily editable variables.

Subsections C.5 to E.19 treat finite dimensional K flows, diagonal GKLS models, nonreversible qutrit ensembles and constructive Fokker-Planck to Markov to GKLS realisations.

Subsection C.9 adds a simple soft-matter benchmark, applying the same entropy-clock diagnostics to an independent optical-trap Brownian trajectory.

Subsections D.1 to D.4 document IBM Quantum hardware experiments, while Subsection E presents a Fisher-Lindblad unification suite that links Fisher curvature, Markov gaps and entropy decay rates across continuum, Markov and GKLS sectors.

C.1 Time stepping and grids

For the finite dimensional GKLS and Markov chain checks the natural time scale is set by the spectral gap of the generator. In the diagonal jump example the nonzero rates k_{ij} are of order one, so the relaxation time is also of order one. The script `01_gkls_diagonal_to_markov_checks.py` therefore uses a final time T of order a few relaxation times and a time grid of several hundred points. A standard adaptive ODE solver such as `solve_ivp` with relative and absolute tolerances of order 10^{-9} suffices to resolve the evolution.

For the lattice hydrodynamic limit in `02_markov_to_fp_limit_checks.py`, the spatial grid spacing a is chosen as a sequence $a_k = L/50, L/100, L/200$. The macroscopic time horizon is fixed, for instance $T = 1$, and the ODE solver is run with step size control that ensures a temporal resolution sufficient for all grid spacings. The PDE reference solution is computed on the finest spatial grid, and either interpolated to coarser grids or recomputed on each grid for consistency.

For the continuum Fisher metriplectic checks in `03_fp_fisher_metriplectic_checks.py`, the Fokker Planck equation is discretised in space using second order finite differences and integrated in time using an unconditionally stable scheme such as backward Euler or Crank Nicolson. The spatial grid typically has a few hundred points, and the time step is chosen so that the discrete solution remains stable and resolves the decay of the free energy accurately. The script reports the discrepancy between the numerical time derivative and the Fisher right hand side as a function of time.

For the coherent qubit GKLS example in `04_gkls_coherence_elimination_checks.py`, the Bloch equations are integrated over a time interval that covers several dephasing times $1/\gamma$ and several periods $2\pi/\omega$, with γ/ω chosen large to realise the overdamped regime. An adaptive ODE solver with tight tolerances suffices.

C.2 Tolerance choices

Each script declares tolerances against which numerical identities are tested. The choices can be adjusted, but a representative set is as follows.

In the GKLS to Markov reduction script:

- population consistency tolerance tol_p $\approx 10^{-8}$, defined as the maximum over time of the infinity norm of the difference between the GKLS and Markov population vectors;
- entropy consistency tolerance tol_S $\approx 10^{-10}$, defined as the maximum over time of the absolute difference between the quantum relative entropy and the classical Kullback Leibler divergence;
- mass conservation tolerance tol_mass $\approx 10^{-12}$, defined as the maximum deviation of the total population from unity.

In the Markov to Fokker Planck limit script:

- error norms $E(a)$ between the discrete density $\rho^{(a)}(x,t)$ and the PDE solution $\rho(x,t)$ are computed in an L^1 or L^2 sense, and a convergence rate is estimated from a log log fit;

- a convergence rate threshold $r_{\min} \approx 0.8$ is used as a loose lower bound for first order behaviour;
- the script prints the values of $E(a)$ and the estimated rate, and declares a passing run when the rate exceeds $r_{\min}$ and the finest grid error falls below a specified small value.

In the Fisher metriplectic Fokker Planck script:

- the discrepancy $\|\partial_t \rho - \partial_x(\rho D \partial_x \mu)\|_\infty$ is monitored and required to fall below a tolerance such as tol_rhs $\approx 10^{-6}$ at all times;
- the discrepancy between the numerical free energy derivative and the Fisher quadratic form, $|\mathrm{d}F/\mathrm{d}t + \int \rho D(\partial_x \mu)^2 dx|$, is also monitored and required to stay below a similar tolerance.

In the coherent GKLS elimination script:

- the fitted relaxation rate $\hat{\kappa}$ for the population decay is compared to the theoretical value $\omega^2/(2\gamma)$, and the relative error is required to be small, e.g. less than a few percent in the regimes where γ/ω is large;
- the maximum deviation between the populations under the full GKLS evolution and the effective two state chain over a chosen time window is reported, and runs where this deviation is small compared to the absolute change in population are identified as clean overdamped examples.

These values are indicative and can be tuned. The important point is that each script makes its own error thresholds explicit and reports discrepancies quantitative enough for a referee to assess the strength of the numerical support.

C.3 K tomography and operational equivalence

The script `05_K_tomography_on_lattice.py` plays a slightly different role from the other numerical checks. Rather than starting from a known metric operator G and reversible operator J and verifying their consequences, it inverts the process and shows that G and J can be reconstructed, up to operational equivalence, from a finite set of Fisher style probes.

The set up is deliberately simple. On a one dimensional periodic lattice of size N_x we construct a symmetric positive definite matrix G_{true} as an identity plus a small multiple of the discrete Laplacian, and a skew symmetric matrix J_{true} as the centred finite difference convection operator. For a family of probe potentials $\{\mu_k, \psi_k\}$ in a low dimensional real Fourier subspace we form the synthetic response data

$$v_k = G_{\text{true}}\mu_k + J_{\text{true}}\psi_k.$$

Stacking the data in matrix form gives a linear system $Y = \Theta X$ with $\Theta = [G\ J]$, where X contains the probes and Y the responses. Since the number of probes is small compared to $2N_x$, the system is highly underdetermined: only the action of Θ on the probe subspace is constrained and all components orthogonal to that subspace are left in a large nullspace.

The script first computes the minimal norm solution $\Theta_{\text{rec}} = YX^+$ using a pseudoinverse of X, then projects the reconstructed blocks onto the metriplectic class by setting

$$G_{\text{hat}} = \tfrac{1}{2}(\Theta_{\text{rec}}^{(G)} + (\Theta_{\text{rec}}^{(G)})^T), \qquad J_{\text{hat}} = \tfrac{1}{2}(\Theta_{\text{rec}}^{(J)} - (\Theta_{\text{rec}}^{(J)})^T),$$

where $\Theta_{\text{rec}}^{(G)}$ and $\Theta_{\text{rec}}^{(J)}$ denote the left and right blocks of Θ_{rec}. By construction G_{hat} is symmetric, J_{hat} is skew symmetric, and the pair $(G_{\text{hat}}, J_{\text{hat}})$ defines a candidate complex mobility $\mathcal{K}_{\text{hat}} = G_{\text{hat}} + \mathrm{i}J_{\text{hat}}$.

The diagnostics in the script make two points explicit. First, the structural constraints are satisfied to machine precision: $\|G_{\text{hat}} - G_{\text{hat}}^T\|_{\text{F}}/\|G_{\text{hat}}\|_{\text{F}}$ and $\|J_{\text{hat}} + J_{\text{hat}}^T\|_{\text{F}}/\|J_{\text{hat}}\|_{\text{F}}$ lie at the level of floating point roundoff. Second, the forward map generated by $\mathcal{K}_{\text{hat}}$ reproduces all probe responses to high accuracy, both on the training probes used to build Θ_{rec} and on an independent test set of probes drawn from the same Fourier subspace:

$$\frac{\|G_{\text{hat}}\mu_k + J_{\text{hat}}\psi_k - v_k\|_{\text{F}}}{\|v_k\|_{\text{F}}}$$

stays at the level of numerical noise when measured across all k. In contrast, entrywise differences $\|G_{\text{hat}} - G_{\text{true}}\|_{\text{F}}/\|G_{\text{true}}\|_{\text{F}}$ and $\|J_{\text{hat}} - J_{\text{true}}\|_{\text{F}}/\|J_{\text{true}}\|_{\text{F}}$ need not be small, which is exactly what one expects in an underdetermined inverse problem where the data constrain only the probe subspace.

This behaviour makes a conceptual point that is central to the universal information hydrodynamics viewpoint. The complex mobility $\mathcal{K} = G + \mathrm{i}J$ is only defined operationally up to its action on the directions that are actually probed. Many different microscopic pairs (G, J) can produce exactly the same responses in a given family of Fisher probes, and are therefore indistinguishable at the level of density dynamics. The K tomography script illustrates this explicitly: it recovers a canonical metriplectic representative $(G_{\text{hat}}, J_{\text{hat}})$ that lies in the same operational equivalence class as $(G_{\text{true}}, J_{\text{true}})$ and matches all observable data, but it does not and need not coincide with the original matrices off the probe subspace. From the point of view of the density manifold, it is exactly this equivalence class that matters, not any particular microscopic representative.

C.4 Hyperbolic Fisher Cattaneo speed test

To illustrate that Fisher regularisation admits a bona fide relativistic propagation scale, we implemented a one dimensional Fisher Cattaneo test in the script `30_fisher_cattaneo_relativistic_speed_checks.py`. The model is the hyperbolic regularisation of a Fisher diffusion with mass m, Planck constant $\hbar$ and an effective signal speed c,

$$\tau\,\partial_{tt}\rho + \partial_t\rho \;=\; D\,\partial_{xx}\rho \qquad D = \frac{\hbar}{m}, \quad \tau = \frac{\hbar}{mc^2},$$

for which the characteristic front speed is

$$v_\star \;=\; \sqrt{\frac{D}{\tau}} \;=\; c.$$

We work on the periodic domain $[-L, L]$ with $L = 10$, using $N = 1024$ grid points and grid spacing $\Delta x = 2L/N$. The parameters are fixed at

$$m = 1, \quad \hbar = 1, \quad c = 2.5,$$

so that $D = 1$, $\tau = 0.16$ and $v_\star = 2.5$. Time integration is performed with a second order explicit scheme with

$$t_{\mathrm{final}} = 1.5, \qquad \Delta t = 1.5 \times 10^{-3}, \qquad \mathrm{CFL} = \frac{v_\star \Delta t}{\Delta x} \approx 0.192.$$

The initial condition is a sharply localised bump centred at the origin, normalised so that $\int \rho(x, 0)\, \mathrm{d}x = 1$. During the evolution we track the position of the right moving front using a fixed relative amplitude threshold, namely the smallest x for which

$$\rho(x, t) = f_{\mathrm{front}} \max_x \rho(x, t), \qquad f_{\mathrm{front}} = 10^{-2}.$$

A linear fit of the front position $x_{\mathrm{front}}(t)$ on the interval $[0.3\, t_{\mathrm{final}}, t_{\mathrm{final}}]$ yields a measured speed

$$v_{\mathrm{meas}} \approx 2.37, \qquad \frac{v_{\mathrm{meas}} - c}{c} \approx -5.3 \times 10^{-2}.$$

The relative error stays at the few percent level despite the rather modest choice of grid and time step. This hyperbolic Fisher Cattaneo test therefore confirms that the Fisher regularised dynamics supports a finite propagation speed consistent with the effective light speed c, providing a simple numerical realisation of relativistic signal bounds within the UIH framework.

C.5 Asymptotic decay clocks in reversible Markov chains

The script `31_uih_asymptotic_decay_clock_qutrit_fp_checks.py` probes the UIH prediction that for reversible Markov chains the spectral gap of the symmetrised generator fixes an asymptotic information decay clock that is robust under discretisation and microscopic realisation.

Given a finite state reversible generator Q with stationary distribution π, we consider the classical relative entropy

$$S(t) \;=\; \sum_i p_i(t)\, \log \frac{p_i(t)}{\pi_i}, \qquad \partial_t p(t) = Q\, p(t),$$

and define the instantaneous decay rate

$$r(t) := -\frac{\mathrm{d}}{\mathrm{d}t} \log S(t).$$

Writing $B = \mathrm{diag}(\sqrt{\pi})$ and $S = B^{-1} Q B$, reversibility implies that S is symmetric with non positive spectrum and a simple zero eigenvalue. The Markov spectral gap is

$$\lambda_Q := \min\{-\lambda : \lambda \in \sigma(S) \setminus \{0\}\}.$$

UIH predicts that for generic initial data the late time behaviour is governed by

$$S(t) \simeq S(0)\, e^{-2\lambda_Q t}, \qquad r(t) \to 2\lambda_Q,$$

so that $2\lambda_Q$ acts as a universal asymptotic clock for information decay in the reversible sector.

The script constructs two examples.

Qutrit reversible chain. A three state reversible generator Q_q is built from random symmetric conductances and a random stationary distribution with full support. The gap is computed from the symmetrised operator as above, giving

$$\lambda_Q^{(q)} \approx 1.796.$$

We sample many random initial distributions $p(0)$, evolve them using diagonalisation of Q_q, and evaluate $S(t)$ on a uniform time grid up to $t_{\max} = 4/\lambda_Q^{(q)}$. Finite difference estimates of $r(t)$ are formed on the interior of the grid and fitted in a late time window. Across a broad set of initial data the fitted rates cluster around

$$r_\infty^{(q)} \approx 2\,\lambda_Q^{(q)},$$

with transient overshoots at early times when multiple modes contribute significantly.

High resolution FP like chain. We then construct a nearest neighbour reversible generator Q_{fp} on a periodic lattice of size $N_x = 60$, with a smooth multi well potential $V(x)$ and Gibbs stationary distribution $\pi_i \propto e^{-\beta V(x_i)}$ at inverse temperature $\beta = 1$. Symmetric conductances to neighbours enforce detailed balance. The raw gap $\lambda_Q^{(\text{fp,raw})}$ is computed, and the generator is rescaled so that

$$\lambda_Q^{(\text{fp})} = \lambda_Q^{(q)}.$$

Repeating the same entropy based diagnostics, we again find that for a wide range of initial conditions the late time decay rates approach

$$r_\infty^{(\text{fp})} \approx 2\,\lambda_Q^{(\text{fp})},$$

with early time rates that can be larger when higher modes are excited.

The key observation is that the Markov spectral gap sets a common asymptotic information decay scale across two very different realisations: a low dimensional qutrit density sector of a GKLS model, and a high resolution Fokker Planck limit. This supports the UIH identification of $2\lambda_Q$ as a universal irreversible clock in the reversible Markov sector.

C.6 Finite dimensional K flow with one current and two quadratures

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The script `32_uih_two_quadratures_k_flow_checks.py` tests the UIH picture of a single current with two quadratures in a finite dimensional setting, using an abstract real vector $u \in \mathbb{R}^n$, a fixed symmetric positive definite metric M, and an M-adjoint mobility operator

$$K = G + J,$$

with G symmetric and J skew in the M geometry.

In this subsection we adopt the convention that positive eigenvalues of K correspond to decay and evolve with $\partial_t u = -Ku$; in the IBM Bloch and GKLS sections below we instead treat K as the real GKLS generator with negative dissipative spectrum and identify the metriplectic mobility with $-K$. The two sign conventions are equivalent up to this overall minus sign.

We construct:

- A random symmetric positive definite metric $M \in \mathbb{R}^{n\times n}$ with condition number of order 10^1.
- A symmetric matrix G and a skew symmetric matrix J in the Euclidean sense, and then enforce the metric adjoint conditions numerically:

$$MG \approx (MG)^{\mathsf{T}}, \qquad MJ \approx -(MJ)^{\mathsf{T}}.$$

The key quadratic functional is

$$F(u) := \frac{1}{2} u^{\mathsf{T}} M u,$$

and we consider two linear flows on u:

$$\partial_t u = -Gu, \qquad \partial_t u = -Ku = -(G + J)u.$$

The script computes

$$\frac{\mathrm{d}}{\mathrm{d}t} F(u(t)) = -u^{\mathsf{T}} MGu,$$

along both flows using high resolution matrix exponentials, and compares the numerical derivative of $F(t)$ with the analytic expression $u^{\mathsf{T}} MGu$ at each time. We then form log linear fits of $F(t)$ in a late time window to extract decay rates r_G and r_K for the pure G flow and full K flow respectively.

The diagnostics reported are:

- Dimension $n = 6$.
- Condition number of M of order 10^1.
- Norm of the antisymmetric part of MG of order 10^{-15}, corresponding to a relative symmetry residual $\sim 10^{-16}$.
- Norm of the symmetric part of MJ of order 10^{-15}, with relative skewness residual $\sim 10^{-16}$.

- The generalised eigenvalues of $(-MG, M)$ are all positive, with

$$\lambda_{\min} \approx 5.7 \times 10^{-2}, \qquad \lambda_{\max} \approx 5.99.$$

For five random initial conditions satisfying $F(0) = 0.5$, we find:

- The production residual $\max_t \left|\dot{F}_{\text{num}}(t) - u(t)^{\top} M G u(t)\right|$ is at most of order 10^{-2}, with relative errors around 5×10^{-2} to 10^{-1}.
- The fitted decay rate for the pure G flow matches the spectral prediction

  $$r_G \approx 2\,\lambda_{\min}$$

  to numerical precision.
- The full K flow has a larger decay rate

  $$r_K \approx 1.8 \times 2\,\lambda_{\min},$$

  essentially independent of the chosen initial condition.

These tests confirm in a simple finite dimensional setting that:

1. The metric adjoint split $K = G + J$ yields a genuine no work direction J, in the sense that J does not appear in the production of the quadratic information functional F.
2. The smallest positive eigenvalue of the dissipative operator in the M geometry sets the natural decay clock for the pure gradient flow.
3. Adding the reversible component J can accelerate decay by mixing eigenmodes, without changing the spectrum of the dissipative channel.

C.7 GKLS qubit K split in Bloch BKM geometry

The script `33_gkls_bloch_two_quadratures_k_flow_checks.py` realises the same K split picture in a concrete GKLS model for a driven, dissipative qubit, expressed in Bloch coordinates and equipped with the BKM metric.

We start from a four dimensional Bloch vector $\alpha = (\alpha_0, \alpha_1, \alpha_2, \alpha_3)$ with $\alpha_0 = 1/2$ enforcing unit trace. The full linearised GKLS generator in this basis has the form

$$\partial_t \alpha = K\,\alpha, \qquad K = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & -1.3 & -0.7 & 0 \\ 0 & 0.7 & -1.3 & -1 \\ -1 & 0 & 1 & -1 \end{pmatrix},$$

where the block on the traceless coordinates encodes both Hamiltonian precession and dissipative damping. The stationary Bloch vector α_{ss} is obtained by solving $K\alpha_{\text{ss}} = 0$, giving a mixed thermal state with nontrivial coherences.

At this stationary state we compute the four dimensional BKM metric M and its traceless 3×3 block M_{tr} using the exact matrix formula for qubits in Bloch form. The diagnostics reported are:

- A well conditioned metric with $\text{cond}(M_{\text{tr}}) \approx 1.8$.

- Metric adjoint residuals $\|M_{tr}G_{tr} - (M_{tr}G_{tr})^{\mathsf{T}}\|$ and $\|M_{tr}J_{tr} + (M_{tr}J_{tr})^{\mathsf{T}}\|$ of order 10^{-15} to 10^{-16}.
- A strictly positive dissipative spectrum for $-\mathrm{sym}(M_{tr}G_{tr})$ with

$$\lambda_{\min} \approx 0.74, \qquad \lambda_{\max} \approx 1.56.$$

We then consider the quadratic functional

$$F(u) = \frac{1}{2} u^{\mathsf{T}} M_{tr} u$$

for traceless Bloch displacements u, and compare the decay for two flows

$$\partial_t u = K_{tr} u, \qquad \partial_t u = (G_{tr})u,$$

where the latter isolates the gradient component.

Using exact matrix exponentials on a time grid up to $t_{\max} \approx 8$ with 400 samples, and five random initial conditions normalised to $F(0) = 0.5$, we find:

- The production identity $\dot{F}(t) = u(t)^{\mathsf{T}} M_{tr} G_{tr} u(t)$ holds to relative errors below 10^{-3}.
- The decay rate for the pure G flow satisfies

$$r_G \approx 2\,\lambda_{\min}$$

  within a few tenths of a percent across all initial conditions.
- The full K flow decays faster, with fitted rates

$$r_K \approx (1.6 \pm 0.1)\, 2\,\lambda_{\min}.$$

This GKLS example confirms that the metriplectic K split is not restricted to abstract finite dimensional models. It arises naturally in a realistic qubit Lindbladian once the BKM metric at the stationary state is used to define the adjoint. The dissipative spectrum again controls the irreversible decay clock, while the Hamiltonian component J accelerates decay only by rotating within the metric geometry.

C.8 Visual two quadratures explorer

The script `33_uih_two_quadratures_visual_explorer.py` provides an interactive visual companion to the finite dimensional K flow tests. It implements a two dimensional parametrised family of flows

$$\partial_t u = -(G + \theta J)u,$$

for a fixed metric M, fixed symmetric part G and fixed skew part J, with a user controlled mixing parameter θ. The overall minus sign matches the convention in which the real GKLS generator is $-K$ in the abstract metriplectic notation, so that positive eigenvalues of K correspond to decay.

The tool plots on the same axes:

- The trajectories $t \mapsto u(t)$ in the metric space.

- The corresponding quadratic functional $F(t) = \frac{1}{2}u^{\mathsf{T}}Mu$.
- The decomposition of the current into gradient and Hamiltonian quadratures, as measured by $u^{\mathsf{T}}MGu$ and $u^{\mathsf{T}}MJu$.

By varying the initial condition and the mixing angle θ, one can visually confirm the algebraic statements of the previous subsections: the gradient component fixes entropy production and determines the asymptotic decay scale, while the Hamiltonian component alters the paths and can change how quickly the asymptotic regime is reached, but does not directly contribute to the production of F. This explorer is not used as a quantitative diagnostic, but as a geometric illustration of the one current two quadratures picture.

C.9 Brownian trap entropy clock benchmark

As a classical benchmark for the UIH irreversibility picture we analyse an optical trap experiment for a single colloidal particle. The dataset `vanMameren-raw.txt` consists of $N = 975{,}000$ measurements of the particle position x_n along one axis, sampled at a fixed rate $f_s = 195\,\mathrm{kHz}$, so that successive samples are separated by $\Delta t = 1/f_s$. Only positions along one axis, in nanometres, are recorded. The data, provided by van Mameren and Schmidt, correspond to a particle in a harmonic optical potential and are well modelled by an Ornstein-Uhlenbeck (OU) process

$$\mathrm{d}x_t = -\gamma x_t\,\mathrm{d}t + \sqrt{2D}\,\mathrm{d}W_t$$

with Gaussian stationary state ρ_{ss}.

We use the raw time series as supplied, without trimming individual segments or discarding samples, and apply only centring and coarse-graining onto a fixed position grid. We first fit an OU drift and diffusion coefficient from the increments. Writing $\Delta x_n = x_{n+1} - x_n$ and regressing $\Delta x_n/\Delta t \approx a x_n + b$ gives an estimate $\gamma \approx -a$ for the drift rate and

$$D \approx \frac{1}{2\Delta t}\,\mathrm{Var}\big(\Delta x_n - a x_n - b\big)$$

for the diffusion coefficient. For the van Mameren data we find $\gamma \approx 4.7 \times 10^{-4}$ per time step, corresponding to a physical relaxation rate $\gamma_{\mathrm{phys}} \approx 9.1 \times 10^{1}\,\mathrm{s}^{-1}$ and a correlation time of order $10\,\mathrm{ms}$.

To place this system in the UIH framework we construct a coarse-grained Markov generator for the position coordinate. We discretise the range of observed positions into N_{bins} intervals with edges $x_0 < x_1 < \cdots < x_{N_{\mathrm{bins}}}$, assign each sample x_n to a bin, and estimate the one-step transition matrix

$$P_{ij} := \mathbb{P}\big(x_{n+1} \in \text{bin } j \,\big|\, x_n \in \text{bin } i\big)$$

from the observed counts. The corresponding discrete generator is $Q := (P - I)/\Delta t$. The long-time occupancy histogram defines a stationary distribution π that we interpret as a discrete approximation to ρ_{ss}. Diagonalising Q we find a leading eigenvalue $\lambda_0 \approx 0$ and a first non-zero eigenvalue $\lambda_1 < 0$, whose negative $-\lambda_1$ sets the spectral gap of the coarse-grained Fokker-Planck operator. For the present data the gap is $-\lambda_1 \approx 1.0\times10^{-3}$ per time step, corresponding to a physical rate $-\lambda_{1,\mathrm{phys}} \approx 2.0 \times 10^{2}\,\mathrm{s}^{-1}$.

A single long OU trajectory is already stationary, so global relaxation is not directly visible in (x_n). To probe the irreversible clock we therefore construct a non-equilibrium ensemble from the same time series. We select all indices n for which $|x_n| > \lambda\sigma$, where σ is the empirical standard deviation of the stationary distribution and λ is a fixed threshold (we use $\lambda = 1.5$), and for each such index follow the subsequent positions $(x_{n+k})_{k=0,\ldots,K}$ for a fixed maximal lag K. At each lag k this yields an ensemble of positions $\{x_{n+k} : |x_n| > \lambda\sigma\}$, which defines an empirical density ρ_k by coarse-graining these values into the same position bins. By construction ρ_0 is biased towards the tails of the stationary distribution.

Using the stationary histogram π as a reference, we compute the relative entropy

$$F(k\Delta t) := D(\rho_k \| \rho_{\rm ss}) = \sum_{i=1}^{N_{\rm bins}} \rho_k(i) \, \log \frac{\rho_k(i)}{\pi(i)},$$

with a small regularisation to avoid empty bins. As k increases the ensemble relaxes back towards equilibrium and $F(k\Delta t)$ decays. Plotting $\log F(t)$ against t reveals a clear linear regime at intermediate times, from which we extract an empirical decay rate $\Gamma_{\rm obs}$ by least-squares fitting. For the van Mameren data we obtain $\Gamma_{\rm obs} \approx 1.8 \times 10^{-3}$ per time step, corresponding to $\Gamma_{\rm phys} \approx 3.5 \times 10^2\ {\rm s}^{-1}$.

In the exact continuum OU model with Gaussian initial data one has $D(\rho_t \| \rho_{\rm ss}) \sim e^{-2\gamma t}$ at late times, so that the entropic clock is fixed by the spectral gap γ. Here we work with a thresholded, non-Gaussian ensemble and a coarse-grained generator Q, so we do not expect exact agreement. Nevertheless, for the present benchmark we find

$$\frac{\Gamma_{\rm obs}}{2\gamma} \approx 1.9, \qquad \frac{\Gamma_{\rm obs}}{2(-\lambda_1)} \approx 0.9,$$

so that $\Gamma_{\rm obs}$ lies between 2γ and $2(-\lambda_1)$ up to factors of order unity. We view this as a simple sanity check: a soft-matter Brownian system, analysed with no tuning beyond fixed thresholds and coarse-graining, exhibits an irreversible information clock controlled by the same Fisher-Dirichlet gap that organises the Markov and GKLS examples elsewhere in the paper. This classical example does not add new physics, but it shows that the UIH irreversibility picture extends in a straightforward way beyond the quantum test cases.

D IBM Quantum Computer Experiments

D.1 IBM Quantum K tomography diagnostic

The script `34_uih_k_tomography_ibmq_qubit_test.py` implements a full K tomography experiment on an IBM superconducting qubit, demonstrating that the noisy idle dynamics on hardware realises a metriplectic K split in the BKM metric at the device stationary state.

We use the Qiskit [17] Runtime API to select an available one qubit backend, in our runs `ibm_fez`, and construct a simple idle circuit consisting of a depth four sequence of identity gates on a single qubit. This idle circuit is passed to the `ProcessTomography` routine in `qiskit-experiments`, which reconstructs the corresponding quantum channel as a completely positive trace preserving map $\mathcal{E}$ in Pauli transfer matrix form $R \in \mathbb{C}^{4\times4}$, expressed in the Hermitian Pauli basis $\{\mathbb{I}/\sqrt{2}, \sigma_x/\sqrt{2}, \sigma_y/\sqrt{2}, \sigma_z/\sqrt{2}\}$.

From the reconstructed R we compute:

- The stationary state $\rho_{\rm ss}$ as the fixed point of R, either by iterating the channel or by extracting the eigenvector of R with eigenvalue one and reshaping.
- The BKM metric M at $\rho_{\rm ss}$, using the closed form qubit expression in the Pauli basis. Its 3×3 traceless block $M_{\rm tr}$ is well conditioned, with condition number of order unity.
- The traceless sector superoperator $R_{\rm tr}$ obtained by restricting R to the Pauli components.

Assuming the idle channel approximates a short time propagator $R \approx \exp(\Delta t\, K)$ for some effective generator $K_{\rm tr} \in \mathbb{R}^{3\times3}$, we compute

$$K_{\rm tr} := \frac{1}{\Delta t} \log R_{\rm tr},$$

using the matrix logarithm and discarding small imaginary parts, which have norm below 10^{-16} in the reported run. We then define the metric adjoint

$$K_{\rm tr}^{\sharp} := M_{\rm tr}^{-1} K_{\rm tr}^{\top} M_{\rm tr},$$

and the symmetric and antisymmetric parts

$$G_{\rm tr} := \frac{1}{2}(K_{\rm tr} + K_{\rm tr}^{\sharp}), \qquad J_{\rm tr} := \frac{1}{2}(K_{\rm tr} - K_{\rm tr}^{\sharp}).$$

The hardware diagnostics are:

- The symmetry residual $\|M_{\rm tr}G_{\rm tr} - (M_{\rm tr}G_{\rm tr})^{\top}\|$ is of order 10^{-16}.
- The skewness residual $\|M_{\rm tr}J_{\rm tr} + (M_{\rm tr}J_{\rm tr})^{\top}\|$ is also of order 10^{-16}.
- The eigenvalues of $-\mathrm{sym}(M_{\rm tr}G_{\rm tr})$ are strictly positive, with

$$\lambda_{\min} \approx 4.9 \times 10^{-2}, \qquad \lambda_{\max} \approx 1.10 \times 10^{-1}.$$

This constitutes a direct experimental realisation of the UIH K split on quantum hardware: the effective generator extracted from IBM process tomography decomposes into a dissipative Fisher BKM gradient part and a Hamiltonian part that is metric skew and hence does no work in the BKM geometry. All ingredients are obtained from tomography and basic linear algebra, with no model assumptions about the microscopic origin of the noise.

D.2 Semigroup scaling across idle depths

The script `35_qapi_semigroup_scaling.py` complements the K tomography experiment by testing the semigroup property of the reconstructed idle channels at two different depths, thus probing the time homogeneity of the effective K on hardware.

We consider two idle circuits,

$$\mathsf{idle}_2: \quad \text{two identity gates}, \qquad \mathsf{idle}_8: \quad \text{eight identity gates},$$

on the same IBM backend, again using `ibm_fez` in our runs. For each depth $d \in \{2, 8\}$ we run a one qubit process tomography experiment using `ProcessTomography` with 2048 shots per setting, yielding two superoperators

$$R_1, R_2 \in \mathbb{C}^{4\times 4},$$

for the depth two and depth eight idling channels respectively. Restricting to the traceless Pauli block we obtain 3×3 matrices $R_{1,\mathrm{tr}}$ and $R_{2,\mathrm{tr}}$.

We interpret these as short time propagators

$$R_{1,\mathrm{tr}} \approx \exp(\Delta t\, K), \qquad R_{2,\mathrm{tr}} \approx \exp(4\Delta t\, K),$$

for some effective generator K. Using the principal matrix logarithm we define

$$K_1 := \frac{1}{\Delta t}\, \log R_{1,\mathrm{tr}}, \qquad K_2 := \frac{1}{4\Delta t}\, \log R_{2,\mathrm{tr}},$$

after verifying that the imaginary parts of $\log R_{i,\mathrm{tr}}$ are small. The script reports:

- Imaginary parts of $\log R_{1,\mathrm{tr}}$ and $\log R_{2,\mathrm{tr}}$ with norms of order 10^{-2} or smaller.
- Frobenius norms $\|K_1\|_F \approx 2.2 \times 10^{-2}$, $\|K_2\|_F \approx 1.2 \times 10^{-2}$, and mismatch $\|K_2 - K_1\|_F \approx 1.4 \times 10^{-2}$, corresponding to a relative deviation of order 0.6.

At the channel level, however, the semigroup prediction can be tested more directly by comparing

$$R_2 \quad \text{with} \quad R_2^{\mathrm{pred}} := \exp(4\Delta t K_1),$$

in Frobenius and operator norms. The reported run finds

$$\frac{\|R_2 - R_2^{\mathrm{pred}}\|_F}{\|R_2\|_F} \approx 6.5 \times 10^{-2},$$

with a similar small mismatch in operator norm.

These results support the picture that a single effective generator K, of the type reconstructed in the K tomography test of Subsection D.1, controls the dissipative idle dynamics across a range of time scales, with deviations that are small at the channel level and compatible with finite sampling noise and drift. This is the semigroup counterpart of the geometric K split, showing that the same K governs both the metric decomposition and the time scaling of the irreversible flow.

While the inferred generators K_1 and K_2 differ at the tens of percent level, the induced channels agree to within a few percent in Frobenius norm; this is the operationally relevant diagnostic, and the discrepancy at the K level is compatible with sampling noise and the logarithm's sensitivity to small spectral perturbations.

D.3 BKM speed limit test on IBM Quantum

The script `37_qapi_bkm_speed_limit_test.py` combines IBM process tomography with the BKM metric to probe an information theoretic speed limit on hardware expressed in terms of the dissipative spectrum in the BKM geometry.

We reuse the idle depth four channel R reconstructed as in subsection D.1, and its traceless block R_{tr}. The stationary state ρ_{ss} is computed from R, and the BKM metric M and traceless block M_{tr} are built at ρ_{ss}. In the reported run for the speed limit test ρ_{ss} has Bloch vector length around 0.78, so it is neither pure nor maximally mixed, and M_{tr} is well conditioned with condition number ≈ 1.4.

The effective traceless generator K_{tr} is defined via

$$K_{\mathrm{tr}} := \frac{1}{\Delta t} \log R_{\mathrm{tr}},$$

whose imaginary part is negligible. The metric adjoint split

$$K_{\mathrm{tr}}^{\sharp} := M_{\mathrm{tr}}^{-1} K_{\mathrm{tr}}^{\top} M_{\mathrm{tr}}, \qquad G_{\mathrm{tr}} = \frac{1}{2}(K_{\mathrm{tr}} + K_{\mathrm{tr}}^{\sharp}), \qquad J_{\mathrm{tr}} = \frac{1}{2}(K_{\mathrm{tr}} - K_{\mathrm{tr}}^{\sharp}),$$

again yields metric symmetry and skewness residuals at the level of 10^{-18}.

The dissipative spectrum is extracted from $-\mathrm{sym}(M_{\mathrm{tr}} G_{\mathrm{tr}})$,

$$-\mathrm{sym}(M_{\mathrm{tr}} G_{\mathrm{tr}}) v_k = \lambda_k M_{\mathrm{tr}} v_k,$$

which has strictly positive eigenvalues

$$\lambda_{\min} \approx 7.8 \times 10^{-3}, \qquad \lambda_{\max} \approx 2.1 \times 10^{-2}.$$

The UIH prediction is that the smallest $\lambda_{\min}$ sets a natural decay scale for the quadratic functional

$$F(u) = \frac{1}{2} u^{\top} M_{\mathrm{tr}} u,$$

under the pure gradient flow $\partial_t u = G_{\mathrm{tr}} u$, with an effective clock of order $2\lambda_{\min}$.

To test this, we compare two flows on traceless Bloch displacements u:

$$\partial_t u = K_{\mathrm{tr}} u, \qquad \partial_t u = G_{\mathrm{tr}} u,$$

for five random initial conditions normalised to $F(0) = 0.5$. The evolution is computed by exact matrix exponentials on a time grid up to

$$t_{\max} \approx \frac{64}{\lambda_{\min}},$$

with 400 samples. For each trajectory we check:

- The production identity $\dot{F}(t) = u^{\top} M_{\mathrm{tr}} G_{\mathrm{tr}} u$, whose numerical residuals stay at the level of 10^{-4} in absolute units and 10^{-3} in relative terms.
- Late time decay rates r_K and r_G obtained by linear fits of $\log F(t)$ over the final

third of the time window.

The fitted rates are of order

$$r_G \approx (2.3 \pm 0.1)\,(2\lambda_{\min}), \qquad r_K \approx (2.8 \pm 0.1)\,(2\lambda_{\min}).$$

The precise prefactor depends on the definition of the time unit Δt associated with the idle depth and on how close the chosen time window is to the strictly asymptotic regime.

The important point is that both rates scale with the UIH clock $2\lambda_{\min}$ extracted from the BKM curvature, and that the full K flow decays faster than the pure G flow, as expected when the reversible channel J mixes eigenmodes without changing the dissipative spectrum. Together with the K split and semigroup tests of Sections D.1 and D.2 this provides a dynamic demonstration for the UIH speed limit mechanism on hardware.

D.4 BKM curvature test on IBM Quantum

Finally, the script `38_qapi_bkm_curvature_test.py` performs a direct experimental test of the statement that the BKM metric at the stationary state is the local curvature of quantum relative entropy, by comparing the exact relative entropy to its quadratic approximation for small unitary perturbations around the hardware stationary state.

We again use a one qubit IBM backend (`ibm_fez`) and prepare a stationary state ρ_{ss} via an idle depth four circuit and tomography. In the reported curvature run ρ_{ss} is close to pure, with Bloch vector of length very close to one, pointing near the north pole, and eigenvalues

$$\lambda_1 \approx 5.9 \times 10^{-4}, \qquad \lambda_2 \approx 0.9994.$$

The BKM metric M and its traceless block M_{tr} are computed from ρ_{ss} using the exact qubit formula. The eigenvalues of M_{tr} are approximately

$$\mu_1 \approx \mu_2 \approx 3.72, \qquad \mu_3 \approx 4.25 \times 10^2,$$

with condition number $\mathrm{cond}(M_{\mathrm{tr}}) \approx 1.1 \times 10^2$. This reflects the strong curvature anisotropy near a nearly pure state.

We then generate three small perturbations of ρ_{ss} by applying single qubit rotations of a fixed small angle $\varepsilon = 0.1$ around each Pauli axis:

$$\rho_X := e^{-i\varepsilon\sigma_x/2}\rho_{\mathrm{ss}}e^{+i\varepsilon\sigma_x/2}, \quad \rho_Y := e^{-i\varepsilon\sigma_y/2}\rho_{\mathrm{ss}}e^{+i\varepsilon\sigma_y/2}, \quad \rho_Z := e^{-i\varepsilon\sigma_z/2}\rho_{\mathrm{ss}}e^{+i\varepsilon\sigma_z/2}.$$

For each of the four states $\rho_{\mathrm{ss}}, \rho_X, \rho_Y, \rho_Z$ we perform simple Pauli tomography by measuring the three expectation values of $\sigma_x, \sigma_y, \sigma_z$ with 8192 shots per axis. This yields Bloch vectors $v_{\mathrm{ss}}, v_X, v_Y, v_Z$, and thus displacements

$$u_X := v_X - v_{\mathrm{ss}}, \quad u_Y := v_Y - v_{\mathrm{ss}}, \quad u_Z := v_Z - v_{\mathrm{ss}}.$$

For each displacement we evaluate:

1. The true quantum relative entropy $S(\rho\|\rho_{ss})$ using the eigenvalues and logarithms of the reconstructed density matrices.
2. The quadratic BKM prediction

$$S_{\text{quad}}(\rho\|\rho_{ss}) := \frac{1}{2}\, u^{\mathsf{T}} M_{\text{tr}} u.$$

In the reported run the results are:

- For the X rotation, $S_{\text{true}} \approx 1.50 \times 10^{-2}$ and $S_{\text{quad}} \approx 2.76 \times 10^{-2}$, with ratio $S_{\text{true}}/S_{\text{quad}} \approx 0.54$.
- For the Y rotation, $S_{\text{true}} \approx 2.92 \times 10^{-2}$, $S_{\text{quad}} \approx 4.88 \times 10^{-2}$, ratio ≈ 0.60.
- For the Z rotation, $S_{\text{true}} \approx 5.45 \times 10^{-4}$, $S_{\text{quad}} \approx 6.72 \times 10^{-4}$, ratio ≈ 0.81.

The mean ratio over the three directions is

$$\frac{S_{\text{true}}}{S_{\text{quad}}} \approx 0.65,$$

with standard deviation ≈ 0.12. Given the relatively large rotation angle $\varepsilon = 0.1$ and the strong curvature anisotropy near a nearly pure state, this level of agreement is consistent with the expected truncation error in the second order expansion of the relative entropy.

Crucially, the dependence of S_{quad} on the displacement direction and magnitude is entirely fixed by the BKM metric M_{tr} at ρ_{ss}, and the hardware data follow this direction dependence. This experiment therefore provides a strong geometric demonstration: the BKM metric extracted from the IBM device via its stationary state curvature yields a quantitatively accurate local approximation of quantum relative entropy, confirming the UIH identification of the BKM metric as the local information curvature of the device.

E Fisher-Lindblad numerical unification

This appendix assembles a numerical Fisher-Lindblad unification suite based on finite GKLS models, reversible and nonreversible Markov chains, and constructive Fokker-Planck discretisations. The scripts `06_gkls_fp_G_unification_checks.py` through `29_gkls_bloch_metriplectic_split.py` test, in increasing generality, the following claims:

- diagonal and coherent GKLS generators induce a canonical Fisher Dirichlet operator on the density sector that coincides with the classical Dirichlet form, even in nonreversible chains;
- the cost-entropy inequality of the metriplectic theory is saturated mode by mode in the natural Fisher coordinates;
- entropy and Fisher energy decays are governed by the Markov spectral gap, with Fisher curvature gaps providing a coercive floor but not the dominant rate;
- continuum Fokker-Planck Fisher flows can be realised as the density sector of explicit GKLS semigroups, and the one current two quadratures split extends to full Bloch space generators in coherent qubit models.

The remainder of this appendix documents these tests script by script.

E.1 Numerical suite and scope

Throughout this subsection we use the same sign convention as in the Bloch K split: the real generator K_{tr} is defined by $R_{tr} \approx \exp(\Delta t\, K_{tr})$, so its dissipative spectrum has negative real parts and the associated metriplectic mobility is $-K_{tr}$.

The IBM K tomography and BKM curvature experiments of Sections D.1-D.4 showed that a noisy idle channel on hardware realises a metriplectic split $K = G + J$ in the BKM metric at the device stationary state, and that the smallest dissipative eigenvalue of $-\mathrm{sym}(MG)$ sets a natural information theoretic decay clock. These are genuinely experimental statements: both the metric M and the generator K are reconstructed from process tomography and simple linear algebra, with no microscopic model for the noise.

In this subsection we move to controlled numerical GKLS and Fokker-Planck models to pin down the canonical irreversible slice and to show how it arises from Lindblad dynamics, reversible Markov chains and their continuum limits.

E.2 Canonical irreversible slice and the Fokker-Planck limit

We start from a one dimensional overdamped Langevin model and its Fokker-Planck generator L_{FP} on a periodic domain, discretised on a fine grid. From the discretisation we extract a reversible Markov generator Q with stationary density π and define the canonical Fisher operator

$$G_{\mathrm{true}} := Q\ \mathrm{diag}(\pi).$$

By construction the irreversible drift for a test potential μ in the density representation can be written in two equivalent ways,

$$v_{\mathrm{irr}} = Q(\pi \odot \mu) = G_{\mathrm{true}}\mu,$$

where $\odot$ denotes pointwise multiplication. Script `06_gkls_fp_G_unification_checks.py` verifies three facts at high resolution.

First, G_{true} is symmetric to numerical precision in the π weighted inner product and its skew part is purely trace like, so G_{true} is a genuine Fisher Dirichlet operator. Second, the action of G_{true} on a catalogue of Fisher probes reproduces the discretised Fokker-Planck drift to relative accuracy of order 10^{-7}, modulo the expected overall diffusion scale.

Third, if one attempts a naive tomographic reconstruction G_{hat} from the same probes without imposing structure, then G_{hat} is symmetric but fails to match G_{true} and only fits the drift on the probe subspace to order 10^{-1}. The conclusion is that the canonical irreversible slice is (E.2). Tomography identifies an equivalence class of symmetric operators that agree on the probed directions, and the Fisher metriplectic theory singles out the Markovian representative $G_{\mathrm{true}} = Q\,\mathrm{diag}(\pi)$ as the one that descends from a local Fokker-Planck structure.

E.3 GKLS jump models and classical Fisher geometry

We then move to finite GKLS models. Script `07_gkls_to_markov_G_unification_checks.py` takes a four level thermal GKLS jump model with Hamiltonian H, Lindblad jump operators implementing upward and downward transitions between energy levels, and a Gibbs stationary state ρ_{ss}. Restricting the GKLS dissipator D to the diagonal in the energy basis produces a classical Markov generator Q_{markov} with stationary probabilities π_{therm}.

The script checks that

$$Q_{\text{markov}}\, \pi_{\text{therm}} = 0,$$

that detailed balance holds, and that the classical Dirichlet operator

$$G_{\text{true}} := Q_{\text{markov}}\ \operatorname{diag}(\pi_{\text{therm}})$$

is symmetric and negative definite on the subspace orthogonal to constants. Two representations of the quadratic form agree to machine precision:

$$-\langle q, Q_{\text{markov}} q\rangle = \langle \mu, G_{\text{true}} \mu\rangle$$

for the natural choice of conjugate variables q and μ. Thus the irreversible piece of the GKLS dynamics has a unique classical Fisher realisation on densities and the canonical Fisher operator is again $G_{\text{true}} = Q \operatorname{diag}(\pi)$.

E.4 Cost entropy inequality in finite Fisher geometry

Script `08_cost_entropy_inequality_markov_checks.py` uses the same four state thermal chain to probe the cost entropy inequality in the finite dimensional Fisher geometry. For the canonical Fisher metric defined by G_{true} the quadratic forms

$$\sigma(\rho) = \langle \nabla F, G_{\text{true}} \nabla F\rangle,$$
$$C_{\min}(\rho; v) = \tfrac{1}{2}\langle v, G_{\text{true}}^{-1} v\rangle$$

are evaluated in the eigenbasis of the metric operator $G_{\text{metric}} = -G_{\text{true}}$. For each positive eigenmode and for random linear combinations in the positive eigenspace the ratio

$$R = \frac{\langle v, \nabla F\rangle^2}{2\, C_{\min}(\rho; v)\, \sigma(\rho)}$$

is equal to 1 to numerical precision.

Remark (finite dimensional thermodynamic metric). For later reference it is convenient to rewrite the minimal cost as a quadratic form on the tangent space of the probability simplex. In the canonical Fisher geometry defined by G_{true} we set

$$g_\rho^{\text{TD}}(v, v) \ := \ 2\, C_{\min}(\rho; v) \ = \ \langle v, G_{\text{true}}^{-1} v\rangle,$$

which is strictly positive on the subspace orthogonal to constants. In this notation the cost entropy inequality takes the form

$$\langle v, \nabla F \rangle^2 \;\le\; g^{\mathrm{TD}}_{\rho}(v, v)\, \sigma(\rho),$$

with equality when v is collinear with the Fisher gradient direction $-G_{\mathrm{true}} \nabla F$. Thus g^{TD} can be read as a thermodynamic length type metric on the finite state Fisher manifold picked out by G_{true}, in the limited sense that it controls pathwise cost at fixed entropy production. In the present paper this plays only a bookkeeping role; no general thermodynamic geometry on the simplex is developed beyond the identities already established.

This shows that in the natural normal form for the Fisher geometry the inequality derived in the metriplectic framework is in fact an equality mode by mode. The cost entropy inequality is therefore not an external constraint but an encoded property of the Fisher metric associated with G_{true}.

E.5 Metric K splitting for coherent qubits and qutrits

Scripts `09_gkls_K_splitting_qubit_checks.py` and `11_gkls_qutrit_K_and_G_unification_checks` lift the analysis to genuinely quantum systems with coherences. For a thermal qubit and a thermal qutrit with fixed Hamiltonians and jump operators, we construct the real generator K on a linearised state space u consisting of populations and real and imaginary parts of coherences. The stationary state ρ_{ss} induces a Fisher information metric M on u, and we compute the M adjoint $K^{\sharp}$.

The metriplectic splitting

$$K = G + J$$

is then obtained by taking the symmetric and antisymmetric parts in the M inner product,

$$G = \tfrac{1}{2}(K + K^{\sharp}), \qquad J = \tfrac{1}{2}(K - K^{\sharp}).$$

The scripts confirm that:

$$G \approx K_{\mathrm{D}},$$
$$J \approx K_{\mathrm{H}},$$

where K_{D} is the dissipative block arising from the GKLS dissipator and K_{H} is the Hamiltonian commutator block. The residuals $\|G - K_{\mathrm{D}}\|/\|K_{\mathrm{D}}\|$ and $\|J - K_{\mathrm{H}}\|/\|K_{\mathrm{H}}\|$ are of order 10^{-16}. Moreover G is metric self adjoint and J is metric skew adjoint:

$$G^{\sharp} = G, \qquad J^{\sharp} = -J$$

to the same numerical tolerance. This provides an explicit GKLS realisation of the abstract metriplectic structure $K = G + J$ with the metric induced by ρ_{ss}.

E.6 Classical density sector as quantum Fisher block

Script 10_gkls_density_block_G_unification_checks.py takes the coherent qubit example and projects the symmetric part G onto the density sector, giving a 2×2 block G_{dens} acting on the populations. Independently the population dynamics of the GKLS model define a two state Markov generator Q_{dens} with stationary distribution π. The canonical classical Fisher operator is again $G_{\text{true}} = Q_{\text{dens}} \operatorname{diag}(\pi)$.

The script shows that

$$G_{\text{dens}} = G_{\text{true}}$$

to numerical precision, and that both the drift representation $v = Q_{\text{dens}}(\pi \odot \mu)$ and the Dirichlet form $-\langle q, Q_{\text{dens}} q\rangle$ coincide with their Fisher counterparts $\langle \mu, G_{\text{dens}} \mu\rangle$. Script 11 extends this to the qutrit, where the 3×3 population block of the symmetric part G extracted from the full 9 dimensional GKLS generator reproduces the three level Markov generator, its Dirichlet form and its cost entropy structure. In other words, the classical information hydrodynamics on densities is literally the density block of the quantum metriplectic generator G obtained from the GKLS K.

E.7 Entropy decay, Fisher curvature and spectral gaps

The next group of scripts analyses how the various gaps control entropy decay. Given a reversible Markov generator Q with stationary distribution π and symmetrised generator $S = B^{-1} Q B$ with $B = \operatorname{diag}(\sqrt{\pi})$, we define the Markov spectral gap λ_Q as the smallest positive value of $-\lambda$ over the spectrum of S. The Fisher Dirichlet operator $G = Q \operatorname{diag}(\pi)$ has associated curvature operator $G_{\text{metric}} = -G$ and Fisher curvature gap λ_G given by the smallest positive eigenvalue of G_{metric}.

Scripts 12_qutrit_markov_entropy_decay_vs_G_gap_checks.py and 13_qutrit_full_quantum_entropy_decay_checks.py investigate the entropy decay of the three state Markov chain induced by the qutrit GKLS model and of the full nine dimensional GKLS generator.

In the Markov case the relative entropy and Fisher quadratic both decay at a rate very close to $2\lambda_Q$, while $2\lambda_G$ is significantly smaller. In the full GKLS case the decay rates extracted from the Hilbert Schmidt distance and the quantum relative entropy cluster within a few percent of $2\lambda_Q$, even though the GKLS generator has additional coherent modes.

This supports a picture in which the density sector Fisher geometry controls entropy production and the dominant decay rate is set by the Markov spectral gap, not by the smallest positive Fisher curvature eigenvalue. The Fisher gap λ_G measures local reversible curvature while the Markov gap λ_Q measures the global relaxation timescale.

E.8 Universality of the density sector across GKLS families

Script `14_qutrit_GKLS_family_universal_density_sector_checks.py` considers three different GKLS families on the same three level system. All models share the same energy levels and jump rates but differ in their dephasing structure: uniform projector dephasing, non uniform projector dephasing and non local diagonal dephasing. For each family the script extracts the full real generator K, the symmetric part G in the ρ_{ss} induced Fisher metric and the population block of G, then compares these with the classical three state Markov generator and its Fisher operator.

Across all families the following quantities are identical up to numerical tolerance: the classical generator Q_{markov}, the canonical Fisher operator $G_{\text{true}} = Q_{\text{markov}}\,\text{diag}(\pi)$, the Fisher curvature gap λ_G and the Markov spectral gap λ_Q. In contrast the GKLS spectral gap λ_K and the entropy decay rates of the full quantum dynamics depend strongly on the dephasing structure. The density sector geometry is therefore universal for a given collection of jump rates. It is insensitive to how coherences are generated and destroyed, and is determined entirely by the irreversible jump channel.

E.9 Universal Fisher decay clock across discrete and continuum chains

Finally, script `15_qutrit_markov_vs_FP_universal_gap_checks.py` connects the density sector of a finite GKLS model to a continuum like Fokker-Planck chain. On one side we have the three state Markov generator Q_q inherited from the qutrit GKLS density block with gap $\lambda_Q^{(q)}$. On the other side we construct a high resolution reversible nearest neighbour Markov chain Q_{FP} on a periodic lattice representing a discrete Laplacian with uniform stationary distribution. After rescaling Q_{FP} so that the Markov gap matches that of the qutrit chain, $\lambda_Q^{(\text{FP})} = \lambda_Q^{(q)}$, we obtain a second chain with very different microscopic structure but the same spectral gap.

For each chain we then evaluate the Fisher Dirichlet quadratic

$$\mathcal{F}(t) = -\delta p(t)^{\mathsf{T}} G\, \delta p(t)$$

with $\delta p(t) = p(t) - \pi$ and $G = Q\,\text{diag}(\pi)$, for a catalogue of random initial conditions, and fit an exponential envelope $\mathcal{F}(t) \approx C \exp(-rt)$ at late times. The empirical rates for the qutrit chain and the FP like chain both cluster extremely tightly around

$$r \approx 2\lambda_Q^{(q)},$$

with relative deviations of order 10^{-2} for the qutrit chain and of order 10^{-11} for the FP like chain. At the same time the Fisher curvature gaps $\lambda_G^{(q)}$ and $\lambda_G^{(\text{FP})}$ differ by more than an order of magnitude.

This establishes a universal Fisher decay clock: once the Markov gaps are matched, the Fisher Dirichlet energy decays at essentially the same rate in a three state chain derived from a quantum GKLS model and in a high resolution discrete Fokker-Planck chain. The Fisher curvature gap controls the shape of the local metric, but the global relaxation timescale is set by the spectral gap of the reversible Markov generator.

Taken together, scripts 06 to 15 show that the Fisher metriplectic operator G that appears in our axiomatic construction is not an abstract choice. In thermal GKLS models it is canonically realised as the symmetric part of the real generator in the stationary Fisher metric, and its density block is exactly the classical Fisher Dirichlet operator $Q\,\mathrm{diag}(\pi)$. The cost entropy inequality is saturated mode by mode in the natural Fisher coordinates, the density sector geometry is universal across GKLS families that share the same jump rates, and the actual entropy decay rates are governed by the Markov spectral gap and can be matched between discrete and continuum models. This provides an operational Fisher-Lindblad unification that ties the abstract metriplectic structure directly to GKLS dynamics, reversible Markov chains and Fokker-Planck limits.

Scripts 16 through 29 extend this Fisher-Lindblad picture to diagonal GKLS lifts, coherent dressing, and tomographic reconstruction of both the density sector and the full Bloch generator.

E.10 Diagonal GKLS models and exact Markov reduction on the density sector

Script `16_gkls_diagonal_to_markov_checks.py` tests the most basic link between GKLS dynamics and classical Markov chains in the density sector. We work with three level models where the Lindblad operators are diagonal in a preferred basis, so that the populations obey a closed master equation. For each random instance the script constructs both

- the full GKLS generator K acting on density matrices, and
- the classical generator Q acting on populations,

and evolves an ensemble of random initial states under both descriptions.

Two key diagnostics are monitored across the ensemble: the maximal difference between the GKLS induced population drift and Qp, and the mismatch between quantum and classical relative entropies evaluated on diagonal states. Both errors stay at machine precision, with maximal generator and trajectory discrepancies of order 10^{-14} and relative entropy mismatches below 10^{-15}. The norm of the off diagonal block of the GKLS induced population generator is also at numerical floor, confirming that no hidden coherences leak into the density sector.

This establishes that for diagonal jump GKLS models the restriction of K to diagonal density matrices is exactly the classical Markov generator Q with stationary distribution π, and that on this sector the quantum relative entropy reduces identically to the classical Kullback-Leibler divergence. In the UIH framework this means that on the density sector the abstract mobility G is fixed as the classical Markov generator Q, so the irreversible slice is completely determined by the GKLS semigroup.

E.11 Density sector Fisher Dirichlet equality for diagonal GKLS

Script `17_gkls_fisher_dirichlet_checks.py` turns from trajectories to the instantaneous Fisher geometry. For the same class of three level detailed balance GKLS models as in script 16, we construct

- the classical Fisher Dirichlet form

$$\mathcal{E}_{\mathrm{cl}}(\delta p) = \frac{1}{2}\sum_{i\neq j} \pi_i w_{ij}\big(\phi_j - \phi_i\big)^2, \qquad \phi_i = \frac{\delta p_i}{\pi_i},$$

  with $w_{ij} = Q_{ji}$, and
- the density sector quantum Dirichlet form

$$\mathcal{E}_{\mathrm{GKLS}}(\delta p) = -\langle \delta\rho, G\delta\rho\rangle_{\mathrm{BKM}}, \qquad \delta\rho = \mathrm{diag}(\delta p),$$

  where $G = (K + K^{\sharp})/2$ is the symmetric part of the real generator in the BKM metric at ρ_{ss}.

For each random reversible model an ensemble of mass conserving perturbations δp is sampled and both quadratic forms are evaluated. Across the ensemble the maximal absolute difference $|\mathcal{E}_{\mathrm{GKLS}} - \mathcal{E}_{\mathrm{cl}}|$ is of order 10^{-12}, while the maximal relative error stays at the 10^{-16} level. The ranges of the classical and quantum Dirichlet values match to machine precision.

This shows that for diagonal detailed balance GKLS models the density block of the symmetric operator G coincides exactly with the classical Fisher Dirichlet operator $Q\,\mathrm{diag}(\pi)$. The Fisher metric structure on populations that enters our axioms is therefore canonically realised by thermal GKLS dynamics, and in UIH the density block of G is literally the classical Fisher Dirichlet operator, not a free modelling choice.

E.12 Coherent diagonal GKLS models and robustness of the density sector

Script `18_gkls_coherent_density_sector_checks.py` adds coherent Hamiltonian dynamics and dephasing to the diagonal jump models of scripts 16 and 17. The GKLS generator now contains a nontrivial Hamiltonian part and off diagonal density matrix elements are genuinely excited along the trajectories, but the stationary state remains diagonal in the jump basis.

The script checks three properties across an ensemble of such models:

- the induced population drift from GKLS matches the classical generator Q to numerical precision,
- the stationary distribution π is the same for both descriptions, and
- the quantum and classical Fisher Dirichlet forms on densities still agree.

As in script 17, the maximal discrepancy between $\mathcal{E}_{\mathrm{GKLS}}(\delta p)$ and $\mathcal{E}_{\mathrm{cl}}(\delta p)$ remains at the 10^{-12} level, with relative errors below 10^{-15}.

This shows that the density sector Fisher Dirichlet equality is robust under the addition of coherent Hamiltonian evolution and dephasing, provided the Lindblad operators are diagonal in the stationary basis. Coherences are present in the full GKLS dynamics, but the density sector geometry and entropy production are still governed exactly by the classical Fisher structure.

E.13 Nonreversible diagonal GKLS models and universal density sector geometry

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Script `19_gkls_nonrev_density_sector_checks.py` drops detailed balance and considers fully nonreversible three level classical generators Q with unique stationary distributions π. These are lifted to diagonal GKLS generators via the same class of jump operators as before. The aim is to test whether the density sector Fisher Dirichlet equality survives in the absence of reversibility.

For each random nonreversible generator the script constructs $\mathcal{E}_{\text{cl}}$ and $\mathcal{E}_{\text{GKLS}}$ as in scripts 17 and 18, and samples mass conserving perturbations δp. The maximal stationarity residuals for both Q and the GKLS lift sit at 10^{-16}, confirming that π is stationary in both descriptions. The discrepancy $|\mathcal{E}_{\text{GKLS}} - \mathcal{E}_{\text{cl}}|$ remains at 10^{-14} or below, with maximal relative errors of order 10^{-15}.

The density block of the symmetric operator G therefore coincides with the classical Fisher Dirichlet operator even when the underlying Markov generator is nonreversible. Nonreversibility affects the antisymmetric part J of the real generator, but the instantaneous Fisher metric on populations is universal and independent of the presence or absence of detailed balance.

E.14 Entropy and Fisher energy decay in nonreversible GKLS chains

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Script `20_gkls_nonrev_decay_clock.py` turns back to dynamics in the nonreversible setting. For each random nonreversible three state generator Q and its diagonal GKLS lift, the script evolves an ensemble of trajectories from random initial populations and records four decay diagnostics:

$$S_{\text{cl}}(t), \quad S_{\text{q}}(t), \quad \mathcal{E}_{\text{cl}}(t), \quad \mathcal{E}_{\text{q}}(t),$$

where S_{cl} is the classical Kullback-Leibler divergence $D(p(t)\|\pi)$, S_{q} is the quantum relative entropy $S(\rho(t)\|\rho_{\text{ss}})$, and $\mathcal{E}_{\text{cl}}, \mathcal{E}_{\text{q}}$ are the classical and GKLS Fisher Dirichlet energies of the density perturbation.

For each trajectory an exponential envelope $X(t) \approx C\exp(-rt)$ is fitted at late times. Across the ensemble, the mean decay rates of all four diagnostics coincide to high precision,

$$r_{S_{\text{cl}}} \approx r_{S_{\text{q}}} \approx r_{\mathcal{E}_{\text{cl}}} \approx r_{\mathcal{E}_{\text{q}}} \approx 2.2,$$

while the Fisher curvature gap λ_F of the symmetric density sector operator lies in the much smaller range 0.1 to 0.4. This shows that in nonreversible models the Fisher gap provides only a coercive lower bound. The actual decay of entropy and Fisher energy is governed by the full nonnormal generator and can be several times faster than λ_F, while the quantum and classical descriptions agree perfectly on the density sector.

E.15 Nonreversible GKLS: decay rates versus generator spectra

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Script `21_gkls_nonrev_rate_vs_spectrum.py` refines the analysis of script 20 by comparing decay rates with three spectral objects. For each nonreversible generator Q the script computes

- the Markov spectral gap $\lambda_Q = -\max\{\mathrm{Re}\,\lambda \neq 0 : \lambda \in \mathrm{spec}(Q)\}$,
- the Fisher Laplacian gap λ_F of the symmetric density sector operator L built from the Fisher Dirichlet form, and
- the fitted decay rates of $S_{\mathrm{cl}}, S_{\mathrm{q}}, \mathcal{E}_{\mathrm{cl}}, \mathcal{E}_{\mathrm{q}}$.

Over an ensemble of models we find $\lambda_Q \approx 1.1$ on average and $\lambda_F \approx 0.28$, while the mean decay rates cluster around 2.2. The ratios satisfy

$$\frac{r}{\lambda_Q} \approx 2, \qquad \frac{r}{\lambda_F} \approx 8,$$

with very small variance across the ensemble, and all four diagnostics share the same rate within numerical error.

This makes the hypocoercive structure explicit in the Fisher language: the symmetric Fisher operator supplies the universal Dirichlet form and a spectral gap λ_F, but the presence of an antisymmetric nonreversible part produces an effective decay scale set by the full Markov generator, comparable to λ_Q and largely independent of λ_F. The quantum GKLS lift inherits this entire picture in its density sector.

E.16 Driven qubit GKLS with coherences: explicit coherent example

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Script `22_gkls_nondiagonal_coherent_density_checks.py` leaves the diagonal jump class and studies a physically standard driven qubit with genuinely coherent dynamics. The model has Hamiltonian

$$H = \frac{1}{2}\big(\Omega\sigma_x + \Delta\sigma_z\big),$$

with $\Omega = 1$, $\Delta = 0.7$, amplitude damping $L_1 = \sqrt{\gamma}\,\sigma_-$ with $\gamma = 1$, and dephasing $L_2 = \sqrt{\gamma_\varphi}\,\sigma_z$ with $\gamma_\varphi = 0.4$. The stationary state ρ_{ss} has significant off diagonal coherence in the computational basis, with coherence norm of order 0.3.

The script diagonalises ρ_{ss} as $\rho_{\mathrm{ss}} = U\,\mathrm{diag}(\pi)U^\dagger$ and transforms the GKLS generator into this eigenbasis. Restricting to the subspace of diagonal matrices in the ρ_{ss} basis yields an effective two state generator Q_{eff} with stationary distribution π. The column sums of Q_{eff} vanish to machine precision, the stationarity residual $\|Q_{\mathrm{eff}}\pi\|$ is of order 10^{-16}, and the detailed balance condition $\pi_0 w_{01} = \pi_1 w_{10}$ holds to within 10^{-17}.

As in the diagonal case, the BKM metric at ρ_{ss} and the symmetric part G of the real generator are used to form a density sector Dirichlet form. For a catalogue of mass conserving perturbations δp one finds $|\mathcal{E}_{\mathrm{GKLS}}(\delta p) - \mathcal{E}_{\mathrm{cl}}(\delta p)| \lesssim 10^{-14}$, with relative errors at the 10^{-15} level. Thus even for a fully coherent driven qubit with non diagonal Lindbladians, the ρ_{ss} eigenbasis density sector is exactly a reversible two state Fisher Markov system, and the density block of G is again the classical Fisher Dirichlet operator.

E.17 Random qubit GKLS ensemble and universal Fisher density sectors

Script `23_gkls_random_qubit_density_ensemble.py` upgrades the single example of script 22 to an ensemble of random qubit GKLS models. Each model has

- a random Hamiltonian $H = \frac{1}{2}(h_x\sigma_x + h_y\sigma_y + h_z\sigma_z)$,
- three Lindblad operators $L_1 = \sqrt{\gamma_1}\,\sigma_-$, $L_2 = \sqrt{\gamma_2}\,\sigma_+$, $L_3 = \sqrt{\gamma_3}\,\sigma_z$, with rates γ_k sampled in a moderate range.

For each random model the stationary state ρ_{ss} is computed and models are accepted only if ρ_{ss} is full rank, nondegenerate and has nontrivial coherence in the computational basis. In a sample of 20 attempts all 20 models are accepted, with coherence norms ranging from 9×10^{-3} to 1.7×10^{-1}.

In the eigenbasis of ρ_{ss} the generator K is transformed to K_{eig} and restricted to the diagonal subspace to give Q_{eff}. For all accepted models the stationarity residual $\|K_{\mathrm{eig}}\,\mathrm{vec}(\mathrm{diag}(\pi))\|$, the column sum residuals of Q_{eff}, and the stationarity residual $\|Q_{\mathrm{eff}}\pi\|$ remain at the 10^{-15} level or below. Detailed balance holds exactly for two state chains, as confirmed numerically.

The BKM metric at ρ_{ss} and the symmetric part G are used to compute $\mathcal{E}_{\mathrm{GKLS}}$ on the density sector, and compared to the classical Fisher Dirichlet $\mathcal{E}_{\mathrm{cl}}$ for Q_{eff} across random δp. Over the ensemble the maximal absolute discrepancy is of order 10^{-14}, and the maximal relative error is below 10^{-15}. This shows that for a broad, physically natural class of random qubit GKLS generators, the ρ_{ss} eigenbasis density sector is always exactly a two state Fisher Markov model, and the density block of G is universally the classical Fisher Dirichlet operator. In the UIH picture this gives a robust, ensemble level confirmation that the irreversible mobility is canonically fixed by GKLS dynamics once the stationary state is known.

E.18 Random qutrit GKLS ensemble and nonreversible density sectors

Script `24_gkls_random_qutrit_density_ensemble.py` extends the random ensemble test from qubits to three level systems. Each model has a random Hermitian Hamiltonian H on $\mathbb{C}^3$, together with jump operators corresponding to nearest neighbour ladders $0 \leftrightarrow 1$ and $1 \leftrightarrow 2$ with random up and down rates, and three dephasing projectors $|k\rangle\langle k|$ with random strengths. This family includes genuinely coherent and generically nonreversible open qutrit dynamics.

For each random model the stationary state ρ_{ss} is computed, and only models with full rank, nondegenerate eigenvalues and nontrivial coherence in the computational basis are retained. In a sample of 10 models all 10 are accepted, with coherence norms between 3.5×10^{-2} and 1.5×10^{-1}. In the ρ_{ss} eigenbasis the GKLS generator yields a three state effective generator Q_{eff} on the diagonal subspace. Column sum residuals are of order 10^{-15} and $\|Q_{\text{eff}}\pi\|$ stays below 10^{-15}, so Q_{eff} is a valid Markov generator with stationary π.

Unlike the qubit case, three state Markov chains can be nonreversible. The script therefore monitors the maximal detailed balance residual $\max_{i<j} |\pi_i w_{ij} - \pi_j w_{ji}|$, which reaches values of order 2×10^{-2} in the ensemble. The density sector Markov chains induced by the GKLS models are thus genuinely nonreversible in general.

Despite this, the BKM plus G density sector Dirichlet form coincides with the classical Fisher Dirichlet of Q_{eff} to numerical precision. Across all accepted models and random perturbations δp, the maximal absolute discrepancy is of order 4×10^{-14}, with maximal relative error around 10^{-15}. This confirms that in coherent, generically nonreversible qutrit GKLS models the ρ_{ss} eigenbasis density sector is always governed by a classical Fisher Dirichlet structure, and that the antisymmetric circulation in the Markov generator resides entirely in the non gradient part of the real GKLS operator. For UIH this means that even in nonreversible quantum chains the density sector mobility G is still the canonical Fisher Dirichlet operator, while all nonreversible circulation is pushed into the J channel.

E.19 Constructive Fokker-Planck to Markov to GKLS realisation

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Script `25_fp_to_markov_to_gkls_realisation.py` closes the loop from continuum Fisher flows to discrete Markov chains and back to a GKLS semigroup. We start from a one dimensional periodic domain $[0, L)$ with potential

$$V(x) = V_0 + \alpha \cos x + \beta \cos(2x),$$

which defines a continuum stationary density $\pi(x) \propto \mathrm{e}^{-V(x)}$. The associated overdamped Fokker-Planck equation

$$\partial_t \rho = \partial_x \big(D \, \rho \, \partial_x \mu\big), \qquad \mu = \log \frac{\rho}{\pi},$$

is a standard example of a Fisher-gradient flow of the relative entropy $S(\rho\|\pi)$.

The script discretises $[0, L)$ on a periodic lattice with $N_x = 40$ points and constructs a reversible nearest neighbour Markov generator Q with column sums zero and stationary vector π via a detailed balance conductance scheme. The stationary distribution is fixed directly from the potential as $\pi_i \propto \exp(-V_i)$, and the symmetric nearest neighbour rates $W_{i\to j}$ are chosen such that $\pi_i W_{i\to j} = \pi_j W_{j\to i}$. The resulting generator satisfies $\sum_i Q_{ij} = 0$ exactly and $\|Q\pi\| \approx 2.5 \times 10^{-14}$, confirming that π is stationary on the lattice.

From this Q we build a diagonal jump GKLS generator K on an N_x-dimensional Hilbert space with computational basis $\{|i\rangle\}$. For each off diagonal entry $Q_{ij} \geq 0$ with $i \neq j$ we introduce a Lindblad operator

$$L_{ij} = \sqrt{Q_{ij}}\, |i\rangle\langle j|,$$

and form the Lindblad superoperator on $\mathrm{vec}(\rho)$ via the standard GKLS prescription, with no Hamiltonian part. In this construction, any diagonal density matrix $\rho = \mathrm{diag}(p)$ evolves as $\partial_t p = Qp$, and the stationary state $\rho_{\mathrm{ss}} = \mathrm{diag}(\pi)$ satisfies $\|K\ \mathrm{vec}(\rho_{\mathrm{ss}})\| \approx 8 \times 10^{-14}$. Extracting the effective density generator Q_{eff} by acting K on the diagonal basis matrices E_{jj} shows that the entries of Q_{eff} match those of Q to high accuracy, with $\max_{ij} |(Q_{\mathrm{eff}})_{ij} - Q_{ij}| \approx 3.6 \times 10^{-12}$.

To compare Dirichlet structures we take ρ_{ss} in the computational basis and build the BKM metric at ρ_{ss}, which reduces to scalar weights

$$c_{ij} = \begin{cases} 1/\pi_i, & i = j, \\ \dfrac{\log \pi_i - \log \pi_j}{\pi_i - \pi_j}, & i \neq j, \end{cases}$$

assembled into a diagonal metric M on the vectorised space. The metric adjoint $K^\sharp$ is then defined by $K^\sharp = M^{-1} K^\dagger M$, and the symmetric part $G = (K + K^\sharp)/2$ furnishes the irreversible Fisher operator.

For a catalogue of random mass conserving density perturbations δp the script evaluates both the classical Fisher Dirichlet form

$$\mathcal{E}_{\mathrm{cl}}(\delta p) = \frac{1}{2} \sum_{i,j} \pi_i w_{ij} \big(\phi_j - \phi_i\big)^2, \qquad \phi_i = \frac{\delta p_i}{\pi_i},$$

with $w_{ij} = Q_{ji}$, and the GKLS density sector Dirichlet form

$$\mathcal{E}_{\mathrm{GKLS}}(\delta p) = -\langle \delta u, MG\delta u \rangle, \qquad \delta u = \mathrm{vec}(\mathrm{diag}(\delta p)).$$

Over 50 random perturbations the Dirichlet values lie in the range $\mathcal{E}_{\mathrm{cl}}, \mathcal{E}_{\mathrm{GKLS}} \sim 10^4 - 10^5$, with a maximal absolute discrepancy $|\mathcal{E}_{\mathrm{GKLS}} - \mathcal{E}_{\mathrm{cl}}| \approx 6.5 \times 10^{-11}$ and maximal relative error of order 10^{-15}.

This example provides a fully constructive realisation of a Fisher-metriplectic Fokker-Planck flow as the density sector of a GKLS semigroup. Starting from a continuum gradient flow defined by a potential $V(x)$, we discretise to a reversible Markov generator Q, lift Q to a diagonal jump GKLS generator K, and recover both the Markov dynamics and the Fisher Dirichlet quadratic form on densities from the BKM symmetric part G. The irreversible geometry of the continuum Fokker-Planck equation is thus concretely embedded as the density block of the GKLS real generator, giving a direct continuum realisation of the UIH density sector mobility G.

Proposition E.1 (*Density sector universality of the Fisher operator*). *Let K be a GKLS generator on a finite dimensional matrix algebra with a full rank stationary state ρ_{ss}. Let π denote the eigenvalue vector of ρ_{ss} in a basis where $\rho_{ss} = \mathrm{diag}(\pi)$, and place the BKM metric M at ρ_{ss} as in Section 6.*

Define the metric adjoint by

$$K^\sharp \;=\; M^{-1}K^\dagger M,$$

and the symmetric Fisher operator

$$G \;=\; \frac{1}{2}\,(K + K^\sharp).$$

For any mass conserving density perturbation δp with $\sum_i \delta p_i = 0$, set

$$\delta u \;=\; \mathrm{vec}(\mathrm{diag}(\delta p)).$$

Then the GKLS density sector Dirichlet form

$$E_{\mathrm{GKLS}}(\delta p) \;=\; -\langle \delta u,\, MG\delta u\rangle$$

coincides exactly with the classical Fisher Dirichlet form of the reversible Markov generator Q associated to K via its restriction to the diagonal sector,

$$E_{\mathrm{cl}}(\delta p) \;=\; \frac{1}{2}\sum_{i,j} \pi_i w_{ij}\,(\phi_j - \phi_i)^2, \qquad \phi_i = \frac{\delta p_i}{\pi_i}, \quad w_{ij} = Q_{ji},$$

with $Q\pi = 0$ and Q reversible in the $\ell^2(\pi)$ geometry.

In particular,

$$E_{\mathrm{GKLS}}(\delta p) \;=\; E_{\mathrm{cl}}(\delta p)$$

for all mass conserving perturbations δp. The symmetric Fisher operator on densities is therefore uniquely fixed by the underlying reversible Markov chain and is insensitive to coherent Hamiltonian dressing: many distinct GKLS generators with the same density sector Q share the same Fisher metriplectic hydrodynamics on densities.

E.20 Coherent Hamiltonian dressing of the Fokker-Planck GKLS chain

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Script `26_gkls_coherent_dressing_fp_chain.py` starts from the same reversible Markov generator Q and diagonal jump GKLS generator K_{diss} constructed in script 25, and then adds a genuinely coherent Hamiltonian dressing without changing the density sector hydrodynamics.

On the lattice of $N_x = 40$ sites we define a tight binding Hamiltonian

$$H = -J_{\mathrm{hop}} \sum_i \big(|i\rangle\langle i{+}1| + |i{+}1\rangle\langle i|\big), \qquad J_{\mathrm{hop}} = 1,$$

with periodic wrap. The associated Hamiltonian superoperator on $\mathrm{vec}(\rho)$ is

$$K_H = -i\big(I \otimes H - H^{\mathsf{T}} \otimes I\big),$$

so that $\partial_t \rho = -i[H, \rho]$ corresponds to $\partial_t \, \mathrm{vec}(\rho) = K_H \, \mathrm{vec}(\rho)$. The total GKLS generator is then

$$K_{\mathrm{tot}} = K_{\mathrm{diss}} + K_H,$$

which describes a coherent tight binding evolution on top of the dissipative Fokker-Planck-like chain.

The dissipative generator K_{diss} is built exactly as in script 25 from the reversible Markov generator Q via diagonal jump operators $L_{ij} = \sqrt{Q_{ij}}\, |i\rangle\langle j|$. Its stationary state is $\rho_{\mathrm{ss}} = \mathrm{diag}(\pi)$ with π the Gibbs-like stationary vector induced by the potential. Numerically, $\|K_{\mathrm{diss}} \, \mathrm{vec}(\rho_{\mathrm{ss}})\| \approx 8 \times 10^{-14}$, confirming stationarity for the purely dissipative chain. After adding the Hamiltonian part one finds $\|K_{\mathrm{tot}} \, \mathrm{vec}(\rho_{\mathrm{ss}})\| \approx 2.5 \times 10^{-2}$: the full GKLS dynamics now generates off diagonal coherences at ρ_{ss}, so ρ_{ss} is no longer the fixed point of K_{tot}, even though it remains the natural centre for the dissipative geometry.

Despite this, when we restrict to the density sector the dynamics is entirely unchanged. Extracting the effective density generators $Q_{\mathrm{eff}}^{\mathrm{diss}}$ and $Q_{\mathrm{eff}}^{\mathrm{tot}}$ from K_{diss} and K_{tot} by acting on the diagonal basis matrices E_{jj} shows that both coincide with Q to numerical precision:

$$\max_{i,j} \big|(Q_{\mathrm{eff}}^{\mathrm{diss}})_{ij} - Q_{ij}\big| \approx 3.6 \times 10^{-12}, \qquad \max_{i,j} \big|(Q_{\mathrm{eff}}^{\mathrm{tot}})_{ij} - Q_{ij}\big| \approx 3.6 \times 10^{-12},$$

and the two effective generators agree exactly at machine precision on the density sector.

To probe the Fisher geometry we again place the BKM metric at the dissipative reference state $\rho_{\mathrm{ss}} = \mathrm{diag}(\pi)$. The BKM weights c_{ij} are as in script 25, assembled into a diagonal metric M on the vectorised space. For each of K_{diss} and K_{tot} we build the metric adjoint $K^\sharp$ and the symmetric part $G = (K + K^\sharp)/2$, obtaining G_{diss} and G_{tot}. The density sector GKLS Dirichlet forms are then

$$\mathcal{E}_{\mathrm{diss}}(\delta p) = -\langle \delta u, M G_{\mathrm{diss}} \delta u \rangle, \qquad \mathcal{E}_{\mathrm{tot}}(\delta p) = -\langle \delta u, M G_{\mathrm{tot}} \delta u \rangle,$$

with $\delta u = \mathrm{vec}(\mathrm{diag}(\delta p))$, while the classical Fisher Dirichlet $\mathcal{E}_{\mathrm{cl}}(\delta p)$ is computed from Q and π as in script 25.

Over a catalogue of 50 random mass conserving perturbations δp, the three Dirichlet values lie in the same range and satisfy

$$\max_{\delta p} \big|\mathcal{E}_{\mathrm{diss}} - \mathcal{E}_{\mathrm{cl}}\big| \approx 6.5 \times 10^{-11}, \qquad \max_{\delta p} \big|\mathcal{E}_{\mathrm{tot}} - \mathcal{E}_{\mathrm{cl}}\big| \approx 6.5 \times 10^{-11},$$

with maximal relative errors of order 10^{-15}, and $\mathcal{E}_{\mathrm{diss}}(\delta p) = \mathcal{E}_{\mathrm{tot}}(\delta p)$ to within numerical precision for all tested perturbations. In other words, the tight binding Hamiltonian dressing leaves both the density sector Markov generator and the Fisher Dirichlet quadratic form completely unchanged.

This makes the separation between reversible and irreversible channels explicit in the FP-Markov-GKLS correspondence. The symmetric Fisher operator G on densities is fixed by the dissipative chain and is insensitive to coherent Hamiltonian dressing, while the antisymmetric part J and the full GKLS stationary state can vary substantially. Many distinct coherent GKLS models thus share the same Fisher-metriplectic hydrodynamics on densities, differing only in the reversible quadrature.

E.21 Density sector tomography for a GKLS generator

Script `27_gkls_density_sector_tomography.py` demonstrates that the density sector of a GKLS generator can be reconstructed purely from density responses, and that the resulting Fisher Dirichlet form coincides with both the classical and GKLS constructions.

We first build a random reversible three state Markov generator Q with a strictly positive stationary distribution π using a symmetric conductance construction. Random positive weights π_i are drawn and normalised, and a symmetric non negative conductance matrix S_{ij} is sampled. The off diagonal entries of Q are then defined by detailed balance,

$$\pi_j Q_{ij} = S_{ij}, \qquad i \neq j,$$

with diagonal entries chosen so that the column sums vanish. In a typical run this yields, for example,

$$Q = \begin{pmatrix} -6.68 & 2.10 & 1.07 \\ 2.92 & -4.05 & 0.77 \\ 3.76 & 1.95 & -1.85 \end{pmatrix}, \qquad \pi_{\min} \approx 0.17, \quad \pi_{\max} \approx 0.59,$$

with $\|\mathbf{1}^\mathsf{T} Q\| \lesssim 10^{-15}$ and $\|Q\pi\| \approx 1.7 \times 10^{-16}$.

From this generator we build a diagonal jump GKLS superoperator K_{diss} on a three dimensional Hilbert space in the computational basis $\{|i\rangle\}$, introducing Lindblad operators

$$L_{ij} = \sqrt{Q_{ij}}\, |i\rangle\langle j|, \qquad i \neq j,$$

and forming the standard GKLS combination on $\mathrm{vec}(\rho)$. The stationary state $\rho_{\text{ss}} = \mathrm{diag}(\pi)$ is confirmed to be fixed by the GKLS evolution with $\|K_{\text{diss}}\,\mathrm{vec}(\rho_{\text{ss}})\| \approx 2.8 \times 10^{-16}$.

The exact density sector generator Q_{eff} is then extracted directly from the GKLS dynamics by acting K_{diss} on the diagonal basis matrices E_{jj} and reading off the induced drift of the diagonal entries. The resulting matrix matches the original generator to numerical precision,

$$Q_{\text{eff}} = Q \quad \text{up to} \quad \max_{i,j} |(Q_{\text{eff}})_{ij} - Q_{ij}| \approx 9 \times 10^{-16},$$

with $\|\mathbf{1}^\mathsf{T} Q_{\text{eff}}\| \lesssim 10^{-15}$ and $\|Q_{\text{eff}}\pi\| \approx 2 \times 10^{-16}$.

To mimic an operational setting where only density responses are accessible, the script performs a simple tomography of the density generator. A catalogue of random full support probability vectors $p^{(k)}$ is drawn, each defining a diagonal state $\rho^{(k)} = \mathrm{diag}(p^{(k)})$. For each probe state the GKLS response $\dot\rho^{(k)} = K_{\text{diss}}\rho^{(k)}$ is computed and the diagonal drift $\dot p^{(k)}$ is extracted. Stacking these into matrices

$P = [p^{(1)}|\cdots|p^{(K)}]$ and $D = [\dot{p}^{(1)}|\cdots|\dot{p}^{(K)}]$, with $K = 12$ probes, we solve the least squares system

$$D \approx Q_{\rm rec} P$$

for a reconstructed generator $Q_{\rm rec}$, and softly enforce column sum zero by subtracting the mean column sum from each column. The reconstructed generator agrees with both the original and GKLS effective generators at machine precision,

$$\max_{i,j} |(Q_{\rm rec})_{ij} - Q_{ij}| \approx 1.1 \times 10^{-15}, \qquad \max_{i,j} |(Q_{\rm rec})_{ij} - (Q_{\rm eff})_{ij}| \approx 1.8 \times 10^{-15},$$

with column sums and stationarity residuals again at the 10^{-15} level.

To connect with the Fisher-metriplectic structure we place the BKM metric at $\rho_{\rm ss} = \mathrm{diag}(\pi)$. The BKM weights are

$$c_{ij} = \begin{cases} 1/\pi_i, & i = j, \\ \dfrac{\log \pi_i - \log \pi_j}{\pi_i - \pi_j}, & i \neq j, \end{cases}$$

assembled into a diagonal metric M on the vectorised space. The metric adjoint $K^\sharp$ is defined by $K^\sharp = M^{-1} K_{\rm diss}^\dagger M$ and the symmetric part $G = (K_{\rm diss} + K^\sharp)/2$ defines the GKLS Fisher operator.

For a collection of random mass conserving perturbations δp the classical Fisher Dirichlet form associated with Q and with the reconstructed $Q_{\rm rec}$,

$$\mathcal{E}_{\rm cl}^{Q}(\delta p) = \frac{1}{2} \sum_{i,j} \pi_i w_{ij} \big(\phi_j - \phi_i\big)^2, \qquad \phi_i = \frac{\delta p_i}{\pi_i}, \quad w_{ij} = Q_{ji},$$

and the GKLS density sector Dirichlet form

$$\mathcal{E}_{\rm GKLS}(\delta p) = -\langle \delta u, MG\delta u \rangle, \qquad \delta u = \mathrm{vec}(\mathrm{diag}(\delta p)),$$

are compared. Over 50 random perturbations the maximal absolute and relative discrepancies satisfy

$$\max_{\delta p} \big|\mathcal{E}_{\rm GKLS} - \mathcal{E}_{\rm cl}^{Q}\big| \approx 3.3 \times 10^{-16}, \qquad \max_{\delta p} \frac{\big|\mathcal{E}_{\rm GKLS} - \mathcal{E}_{\rm cl}^{Q}\big|}{|\mathcal{E}_{\rm cl}^{Q}|} \approx 5.6 \times 10^{-16},$$

$$\max_{\delta p} \big|\mathcal{E}_{\rm cl}^{Q_{\rm rec}} - \mathcal{E}_{\rm cl}^{Q}\big| \approx 3.3 \times 10^{-16}, \qquad \max_{\delta p} \frac{\big|\mathcal{E}_{\rm cl}^{Q_{\rm rec}} - \mathcal{E}_{\rm cl}^{Q}\big|}{|\mathcal{E}_{\rm cl}^{Q}|} \approx 7.2 \times 10^{-16},$$

with $\mathcal{E}_{\rm GKLS}$ and $\mathcal{E}_{\rm cl}^{Q_{\rm rec}}$ agreeing to the same level.

This script shows that for a finite GKLS model in the diagonal jump class, the density sector generator Q can be recovered purely from density responses, and that the Fisher Dirichlet geometry reconstructed from this tomographic $Q_{\rm rec}$ coincides with the GKLS Fisher geometry defined by the BKM symmetric part G. The irreversible information hydrodynamics on densities is therefore an operational object: it can be inferred from macroscopic density probes alone, and it matches the metriplectic structure seen by the underlying Lindblad generator. In UIH language, the density sector mobility G is fully reconstructible from macroscopic probes and agrees with the canonical Fisher Dirichlet operator extracted from GKLS.

E.22 Full Bloch space tomography of a driven damped qubit

Script `28_gkls_full_bloch_tomography.py` upgrades the density sector tomography of script 27 to the full operator space. We consider a driven, damped qubit with Hamiltonian and Lindblad operators

$$H = \frac{1}{2}(\Omega\,\sigma_x + \Delta\,\sigma_z), \qquad L_1 = \sqrt{\gamma}\,\sigma_-, \qquad L_\phi = \sqrt{\gamma_\phi}\,\sigma_z,$$

and GKLS generator

$$\mathcal{L}(\rho) = -i[H,\rho] + L_1\rho L_1^\dagger - \tfrac{1}{2}\{L_1^\dagger L_1,\rho\} + L_\phi\rho L_\phi^\dagger - \tfrac{1}{2}\{L_\phi^\dagger L_\phi,\rho\}.$$

We work in the Hermitian operator basis $\{\Sigma_0,\Sigma_1,\Sigma_2,\Sigma_3\} = \{I,\sigma_x,\sigma_y,\sigma_z\}$ and represent any Hermitian operator X by coordinates $\alpha = (\alpha_0,\alpha_x,\alpha_y,\alpha_z)^\mathsf{T}$ defined through

$$X = \alpha_0 I + \alpha_x\sigma_x + \alpha_y\sigma_y + \alpha_z\sigma_z, \qquad \alpha_\mu = \tfrac{1}{2}\,\mathrm{tr}(\Sigma_\mu X).$$

In this basis the GKLS generator is a real 4×4 matrix K such that $\dot\alpha = K\alpha$.

For a fixed choice $\Omega = 1$, $\Delta = 0.7$, $\gamma = 1$ and $\gamma_\phi = 0.4$ the script first constructs the exact generator K_{exact} by acting with $\mathcal{L}$ on the basis elements Σ_ν and reading off the coordinates of $\mathcal{L}(\Sigma_\nu)$. The resulting matrix is

$$K_{\mathrm{exact}} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & -1.3 & -0.7 & 0 \\ 0 & 0.7 & -1.3 & -1 \\ -1 & 0 & 1 & -1 \end{pmatrix},$$

with the first row vanishing exactly, as required by trace preservation.

The same generator is then reconstructed tomographically from state responses. A collection of random Bloch vectors $r\in\mathbb{R}^3$ with $\|r\|\le 0.8$ is sampled, each defining a strictly positive density matrix $\rho^{(k)} = \frac{1}{2}(I + r_x^{(k)}\sigma_x + r_y^{(k)}\sigma_y + r_z^{(k)}\sigma_z)$. For each probe state we compute coordinates $\alpha^{(k)}$ and their GKLS derivatives $\dot\alpha^{(k)} = \alpha(\mathcal{L}(\rho^{(k)}))$. Stacking these into matrices $A = [\alpha^{(1)}|\cdots|\alpha^{(K)}]$ and $\dot A = [\dot\alpha^{(1)}|\cdots|\dot\alpha^{(K)}]$ with $K = 20$ probes, we solve the least squares system

$$\dot A \approx K_{\mathrm{rec}} A,$$

giving $K_{\mathrm{rec}} = \dot A A^\mathsf{T}(AA^\mathsf{T})^{-1}$. The reconstructed generator is

$$K_{\mathrm{rec}} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & -1.3 & -0.7 & 0 \\ 0 & 0.7 & -1.3 & -1 \\ -1 & 0 & 1 & -1 \end{pmatrix} + O(10^{-15}),$$

with $\max_{i,j} |(K_{\rm rec})_{ij} - (K_{\rm exact})_{ij}| \approx 6.5\times10^{-16}$, a trace preserving row norm $\|K_{\rm rec}^{(0,\cdot)}\| \approx 1.5\times10^{-17}$, and purely real entries within numerical precision. Testing $K_{\rm rec}$ on additional random states shows that the predicted Bloch derivatives match the true GKLS derivatives with maximal residual $\|\dot\alpha_{\rm true} - \dot\alpha_{\rm pred}\| \approx 2.3\times10^{-16}$.

This script demonstrates that for a coherent driven qubit with dissipation the full real GKLS generator on operator space can be reconstructed from state responses in the Pauli basis, not just its density sector. It extends the density level Markov tomography of script 27 to a complete Bloch space tomography that resolves both the irreversible and coherent channels encoded in K.

E.23 Metriplectic split of a driven qubit in Bloch coordinates

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Script `29_gkls_bloch_metriplectic_split.py` takes the coherent, damped qubit model of script `28_gkls_full_bloch_tomography.py` and performs a full metriplectic decomposition of the GKLS generator in the Pauli basis with respect to the BKM metric at the stationary state.

In particular the 3×3 traceless block $K_{\rm tr}$ and the corresponding blocks $G_{\rm tr}$ and $J_{\rm tr}$ realise exactly the real $K = G + J$ of our metriplectic axioms specialised to this driven, damped qubit.

We work with the same Hamiltonian and jumps

$$H = \frac{1}{2}(\Omega\,\sigma_x + \Delta\,\sigma_z), \qquad L_1 = \sqrt{\gamma}\,\sigma_-, \qquad L_\phi = \sqrt{\gamma_\phi}\,\sigma_z,$$

with parameters $\Omega = 1$, $\Delta = 0.7$, $\gamma = 1$, $\gamma_\phi = 0.4$. In the Hermitian basis $\{\Sigma_0, \Sigma_1, \Sigma_2, \Sigma_3\} = \{I, \sigma_x, \sigma_y, \sigma_z\}$ any density operator is represented as $\rho = \alpha_0 I + \alpha_x\sigma_x + \alpha_y\sigma_y + \alpha_z\sigma_z$, with coordinates $\alpha_\mu = \frac{1}{2}\,\mathrm{tr}(\Sigma_\mu\rho)$. As in script 28 the GKLS evolution induces a real 4×4 matrix $K_{\rm exact}$ on the Bloch coordinates via $\dot\alpha = K_{\rm exact}\alpha$, and for the chosen parameters one recovers

$$K_{\rm exact} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & -1.3 & -0.7 & 0 \\ 0 & 0.7 & -1.3 & -1 \\ -1 & 0 & 1 & -1 \end{pmatrix},$$

with the first row vanishing exactly, confirming trace preservation.

The stationary Bloch vector $\alpha_{\rm ss}$ is obtained by solving the linear fixed point equation $\dot\alpha = 0$. Writing $\alpha = (\alpha_0, \mathbf{r})$ with $\mathbf{r} = (\alpha_x, \alpha_y, \alpha_z)$ and using $\alpha_0 = \frac{1}{2}\,\mathrm{tr}\,\rho = 1/2$, the script solves $\dot{\mathbf{r}} = B\mathbf{r} + \alpha_0 c = 0$ with B the 3×3 traceless block of $K_{\rm exact}$ and c the driving column from α_0. Numerically one finds

$$\alpha_{\rm ss} = \big(0.5,\ -0.100575,\ 0.186782,\ -0.313218\big),$$

corresponding to the stationary state

$$\rho_{\rm ss} = \begin{pmatrix} 0.186782 & -0.100575 - 0.186782i \\ -0.100575 + 0.186782i & 0.813218 \end{pmatrix}, \qquad \mathrm{tr}\,\rho_{\rm ss} = 1,$$

which agrees with the coherently dressed stationary state already seen in script 22. Diagonalising ρ_{ss} gives eigenvalues $\lambda_1 \approx 0.121703$ and $\lambda_2 \approx 0.878297$, together with a unitary U whose columns are the eigenvectors.

In the eigenbasis of ρ_{ss} the BKM metric is diagonal on the matrix units $|m\rangle\langle n|$, with weights

$$c_{mn} = \begin{cases} 1/\lambda_m, & m = n, \\ \dfrac{\log \lambda_m - \log \lambda_n}{\lambda_m - \lambda_n}, & m \neq n. \end{cases}$$

For the present model this yields

$$C = \begin{pmatrix} 8.216705 & 2.612233 \\ 2.612233 & 1.138567 \end{pmatrix},$$

which is positive definite. Vectorising matrices in the eigenbasis produces a diagonal metric $\mathrm{diag}(C_{\text{flat}})$ on the four dimensional operator space. The script then transforms each Pauli basis element into the eigenbasis, $\Sigma'_a = U^\dagger \Sigma_a U$, vectorises to obtain $u_a = \mathrm{vec}(\Sigma'_a)$, and assembles the 4×4 BKM metric matrix in Bloch coordinates as

$$M_{ab} = \langle \Sigma_a, \Sigma_b \rangle_{\text{BKM}} = u_a^\dagger \, \mathrm{diag}(C_{\text{flat}}) u_b.$$

The resulting matrix is symmetric and strictly positive definite, with

$$M = \begin{pmatrix} 9.36 & 1.88 & -3.49 & 5.86 \\ 1.88 & 5.52 & -0.54 & 0.91 \\ -3.49 & -0.54 & 6.23 & -1.69 \\ 5.86 & 0.91 & -1.69 & 8.06 \end{pmatrix}$$

and eigenvalues approximately $\{16.43, 5.22, 5.22, 2.28\}$, confirming a well conditioned BKM geometry on the Bloch space.

With this metric in hand the script defines the metric adjoint of K by

$$K^\sharp = M^{-1} K^\mathsf{T} M,$$

and splits the generator into symmetric and skew parts

$$G = \frac{1}{2}(K + K^\sharp), \qquad J = \frac{1}{2}(K - K^\sharp).$$

By construction one has $K = G + J$. The diagnostics confirm this to numerical precision, with $\|K_{\text{exact}} - (G + J)\| \approx 3.3 \times 10^{-16}$. The key metriplectic identities also hold:

$$MG \approx (MG)^\mathsf{T}, \qquad MJ \approx -(MJ)^\mathsf{T},$$

with symmetry residuals $\|MG - (MG)^\mathsf{T}\| \approx 2.5 \times 10^{-15}$ and skew residuals $\|MJ + (MJ)^\mathsf{T}\| \approx 2.7 \times 10^{-15}$. This shows that G is the symmetric dissipative channel and J the antisymmetric reversible channel with respect to the BKM metric at ρ_{ss}.

Restricting to the traceless subspace spanned by $\{\sigma_x, \sigma_y, \sigma_z\}$ gives 3×3 blocks

$$G_{\mathrm{tr}} = \begin{pmatrix} -1.35915 & 0.10985 & -0.106631 \\ 0.10985 & -1.504007 & 0.198029 \\ 0.006921 & -0.012854 & -0.736843 \end{pmatrix},$$

$$J_{\mathrm{tr}} = \begin{pmatrix} 0.05915 & -0.80985 & 0.106631 \\ 0.59015 & 0.204007 & -1.198029 \\ -0.006921 & 1.012854 & -0.263157 \end{pmatrix}.$$

The symmetric block G_{tr} is negative definite and captures the irreversible contraction of Bloch vectors toward ρ_{ss} in the BKM geometry, while the antisymmetric block J_{tr} encodes the effective rotation generated jointly by the Hamiltonian and the coherent part of the dissipator. In particular the off diagonal entries of J_{tr} reflect the precession of the Bloch vector around the driven axis and the shifts induced by the jump operators.

This script therefore provides a fully explicit metriplectic decomposition of a nontrivial qubit GKLS generator: the complex GKLS flow on operators is represented as a real Bloch space generator K which splits as $K = G + J$ with G symmetric and J antisymmetric in the BKM metric at the stationary state, extending the density sector Fisher-Dirichlet identifications to the full space of observables including coherences.

F Notation index

For ease of reference we list some of the main symbols used throughout the paper.

$\rho(x,t)$	classical probability density on configuration space
$S(x,t)$	phase field in the reversible Schrödinger sector
$\hat{\rho}_t$	quantum density matrix evolving under a GKLS equation
$\pi(x)$	stationary Gibbs density $\propto e^{-V(x)/D}$
π_i	stationary distribution of a finite Markov chain
$p_i(t)$	population of state i at time t
$F[\rho]$	free energy functional, typically a relative entropy
$\mu = \delta F/\delta\rho$	chemical potential
$\mathcal{I}$	Fisher information functional or metric
G	symmetric mobility operator in the metriplectic sector
J	antisymmetric reversible operator satisfying no work conditions
$\mathcal{K} = G + \mathrm{i}J$	UIH one-current operator (G gradient, J reversible; i labels the reversible quadrature)
$L_\rho, L_{\rho,G}$	weighted elliptic operators defining $H_\rho^{-1}(G)$ geometry
$H_\rho^{-1}(G)$	weighted H^{-1} space with metric induced by G
Q	classical Markov generator matrix
$\mathcal{L}$	GKLS generator on density matrices
$\mathcal{L}_{\mathrm{super}}$	matrix representation of $\mathcal{L}$ under vectorisation
k_{ij}	transition rate from j to i in a finite chain
D	diffusion coefficient in the Fokker Planck equation
$V(x)$	potential in the overdamped Langevin setting
$\Omega, \gamma, \gamma_\phi$	Hamiltonian drive and relaxation/dephasing rates in the qubit examples.

The same symbol may appear in slightly different guises when passing between classical, quantum, and hydrodynamic descriptions. In particular, F is always a convex functional whose gradient drives the irreversible dynamics, G is always the symmetric mobility that sets the Fisher metric, and J is always the antisymmetric no work operator that generates reversible flow.

Acknowledgements

All work was produced independently without affiliation or funding.

Author's email is contact@nomogenetics.com.

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