Observation Kernels and Hidden Completion Classes

Universal Information Hydrodynamics:
Fisher backbones, slip-silent sectors, and stochastic weak-field floors

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Abstract

We formulate a weak-field quotient picture for Fisher-geometric gravity in which the rigid object is the observed symmetric Fisher backbone, while hidden reversible and stress-bearing structure is identifiable only modulo the kernel of an observation map. For the Kähler current module, scalar slip does not reconstruct the hidden anisotropic sector uniquely: at each nonzero Fourier mode the hidden symmetric trace-free sector splits into one scalar-visible direction, two vector-visible directions, and a two-dimensional tensor reservoir that is invisible to scalar and divergence diagnostics at linear order. In the isotropic large-N regime induced by the current-builder, the moment defect has exact covariance proportional to the projector onto the five-dimensional symmetric trace-free representation, yielding universal visibility fractions $1/5$, $2/5$, and $2/5$ for scalar, vector, and tensor sectors respectively. The resulting current-induced slip field has a universal quadrupolar angular covariance kernel and an N^{-1} spectral floor after good coarse-graining. When combined with the deterministic scalar baseline, the theory predicts that upper masked quantiles bend away from the clean $(1+f)^{-1}$ law before the median, with a crossover scale controlled by the scalar baseline, the current multiplicity N, and a morphology-dependent spectral mixing functional. The note isolates the mathematically hard core of the hidden-completion picture and separates it from stronger extensions that remain to be frozen.

1 Purpose and scope

This note isolates one technical claim from the broader UIH programme. The claim is that the weak-field gravity closure is not naturally a theory of one uniquely reconstructible hidden completion. It is more naturally a theory of a rigid observed Fisher backbone together with a hidden completion class, quotiented by the kernel of the chosen observation map.

Two pieces of prior structure are taken as input.

First, the UIH and Converse Madelung frameworks split a linearised generator into metric-symmetric and metric-skew parts with respect to a Fisher or BKM metric,

$$K = G + J, \qquad G = \frac{1}{2}(K + K^\sharp), \qquad J = \frac{1}{2}(K - K^\sharp), \tag{1}$$

where $K^\sharp = M^{-1}K^\dagger M$. The density-side reversible sector is not arbitrary skewness but the weighted-Liouville no-work cone. Second, the weak-field gravity closure introduces a Kähler current module with density loading

$$\rho_J = f\rho_s, \tag{2}$$

and small coherent traceless stress generated from a moment defect field. In the isotropy-dominated regime the solver empirically finds

$$Q_p(R) \propto (1+f)^{-1} \tag{3}$$

for masked slip ratios, up to a weak floor at large f.

The results below show that these are not separate stories. The weak-field current module realises the quotient structure explicitly.

2 Set-up

Abstract metriplectic background

Let V be a finite-dimensional real linearised state space equipped with a positive metric M. The metric adjoint is

$$A^\sharp := M^{-1}A^\top M. \tag{4}$$

Given a generator K, define the symmetric and skew parts by

$$G := \frac{1}{2}(K + K^\sharp), \qquad J := \frac{1}{2}(K - K^\sharp). \tag{5}$$

Then $G^\sharp = G$ and $J^\sharp = -J$.

In the density realisation, the reversible local class acts on a potential μ by

$$v_A(\mu) = \nabla\cdot(\rho A\nabla\mu), \qquad A^\top = -A, \qquad \nabla_i(\rho A_{ij}) = 0, \tag{6}$$

so that the reversible sector does no work:

$$\int \mu\, v_A(\mu)\,\mathrm{d}x = 0. \tag{7}$$

This is the physically admissible transport class behind the abstract metric-skew gauge freedom.

Weak-field gravity closure

In the quasi-static periodic regime, the scalar potentials satisfy

$$\nabla^2\Psi = 4\pi G_N\,\rho, \qquad \nabla^4 S = 12\pi G_N\,D^{ij}\Pi_{ij}, \tag{8}$$

with

$$\rho = \rho_s + \rho_J, \qquad \rho_J = f\rho_s, \qquad \Pi_{ij} = \Pi_{ij}^{(s)} + \Pi_{ij}^{(J)}. \tag{9}$$

The current module is built from an empirical moment defect

$$\Delta M_{ij} := \frac{1}{N}\sum_{a=1}^{N}\left(u_i^{(a)}u_j^{(a)} - \frac{1}{3}\delta_{ij}\right), \tag{10}$$

so that

$$\Pi_{ij}^{(J)} = \rho_J\,\Delta M_{ij}. \tag{11}$$

The vectors $u^{(a)}$ are pointwise normalised filtered divergence-free Gaussian fields. In the isotropy-dominated regime,

$$\mathbb{E}[u_iu_j] = \frac{1}{3}\delta_{ij}, \qquad \mathbb{E}\,\|\Delta M\|_F^2 = \frac{2}{3N}, \tag{12}$$

and the stress fraction obeys

$$\frac{\|\Pi_J\|}{\rho_J} \sim \|\Delta M\| \sim \varepsilon(\ell,N), \qquad \varepsilon(\ell,N)\sim N^{-1/2}, \tag{13}$$

with further suppression as the physical filter scale increases.

3 Observation kernels and quotient structure

Definition 1 (Observation map)**.** *Let $\mathcal{H}$ denote the hidden sector under consideration. An observation map is a linear or linearised map*

$$\mathcal{O}:\mathcal{H}\to\mathcal{Y} \tag{14}$$

from hidden fields to observed weak-field data.

The observational content of the hidden sector is not the whole of $\mathcal{H}$ but the quotient by the kernel of $\mathcal{O}$.

Theorem 2 (Observation-kernel quotient)**.** *Let $h_1, h_2 \in \mathcal{H}$. Then the observation map identifies them if and only if*

$$\mathcal{O}(h_1 - h_2) = 0. \tag{15}$$

Hence the observed theory reconstructs the equivalence class

$$[h]_{\mathcal{O}} = h + \ker\mathcal{O}, \tag{16}$$

not a unique hidden completion.

Proof. This is immediate from linearity. The same argument works for a linearisation of a nonlinear observation map around a fixed background. □

In the present weak-field setting there are at least two basic observation maps. The density-loading map is

$$\mathcal{P}(\rho_J) = \Psi_J, \qquad \nabla^2\Psi_J = 4\pi G_N\rho_J, \tag{17}$$

and the scalar-slip map is

$$\mathcal{S}_{\rho_J}(\Delta M) = S_J, \qquad \nabla^4 S_J = 12\pi G_N D^{ij}(\rho_J\Delta M_{ij}). \tag{18}$$

The first depends only on the scalar density. The second depends only on the scalar quadrupolar contraction of the symmetric trace-free stress field.

4 Modewise STF decomposition and observation filtration

Fix a nonzero Fourier mode $k\in\mathbb{R}^3$ and let $n = k/|k|$. The space of symmetric trace-free tensors at that mode is five-dimensional. Choose an orthonormal STF basis

$$\{S, V_1, V_2, T_+, T_\times\}, \tag{19}$$

adapted to n, where S is the scalar longitudinal quadrupole, V_1, V_2 are the two vector-type STF modes, and $T_+, T_\times$ are the transverse-traceless modes.

Any hidden STF mode can be written uniquely as

$$\widehat{T}_{ij}(k) = a\,S_{ij} + b_1\,V_{1,ij} + b_2\,V_{2,ij} + c_+\,T_{+,ij} + c_\times\,T_{\times,ij}. \tag{20}$$

Theorem 3 (STF observation filtration)**.** *For each nonzero Fourier mode:*

1. *The scalar slip observable sees only the scalar coefficient a.*

2. *The divergence channel $ik^j\widehat{T}_{ij}$ sees a, b_1, b_2 and is blind to $c_+, c_\times$.*

3. The remaining two TT coefficients require tensor-sensitive observables.

Equivalently,

$$\ker \mathcal{S}_k = \operatorname{span}\{V_1, V_2, T_+, T_\times\}, \tag{21}$$

and

$$\ker(\nabla\cdot)_k = \operatorname{span}\{T_+, T_\times\}. \tag{22}$$

Proof. The scalar slip source contracts the STF stress with $n_i n_j$, hence only the scalar basis tensor contributes. The divergence channel contracts with one factor of k, so the TT modes vanish by transversality while the scalar and vector-type modes remain. The decomposition is orthogonal and exhaustive. □

The hidden current sector is therefore peeled away in stages by observables:

$$\text{density} \to \text{scalar STF} \to \text{scalar plus vector STF} \to \text{full STF}. \tag{23}$$

This will matter both for identifiability and for stochastic floors.

5 Universal isotropic statistics on STF space

Let

$$A_{ij} := u_i u_j - \frac{1}{3}\delta_{ij} \tag{24}$$

for an isotropic unit vector u. Rotational invariance forces the covariance to lie along the projector onto the STF representation.

Lemma 4 (Single-sample STF covariance)**.** *Let*

$$P^{\mathrm{STF}}_{ij,kl} := \frac{1}{2}(\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk}) - \frac{1}{3}\delta_{ij}\delta_{kl}. \tag{25}$$

Then

$$\mathbb{E}[A_{ij}A_{kl}] = \frac{2}{15}P^{\mathrm{STF}}_{ij,kl}. \tag{26}$$

Proof. By isotropy, $\mathbb{E}[A_{ij}A_{kl}] = \alpha P^{\mathrm{STF}}_{ij,kl}$ for some scalar α. Contracting with $\delta_{ik}\delta_{jl}$ and using $\|A\|_F^2 = 2/3$ gives $5\alpha = 2/3$, hence $\alpha = 2/15$. □

Theorem 5 (Moment-defect covariance)**.** *For*

$$\Delta M_{ij} = \frac{1}{N}\sum_{a=1}^{N} A^{(a)}_{ij}, \tag{27}$$

with independent isotropic samples,

$$\operatorname{Cov}(\Delta M_{ij}, \Delta M_{kl}) = \frac{2}{15N}P^{\mathrm{STF}}_{ij,kl}. \tag{28}$$

Proof. Independence removes cross terms, so the covariance scales by $1/N$. □

This fixes the full hidden covariance on STF space, not just its trace.

Universal visibility fractions

In the adapted orthonormal STF basis,

$$\Delta M = \xi_S S + \xi_{V_1}V_1 + \xi_{V_2}V_2 + \xi_+ T_+ + \xi_\times T_\times, \tag{29}$$

and by the theorem above,

$$\mathbb{E}[\xi_A \xi_B] = \frac{2}{15N}\delta_{AB}. \tag{30}$$

Hence the expected power fractions are universal:

$$\boxed{\text{scalar-visible fraction} = \frac{1}{5}, \qquad \text{vector-visible fraction} = \frac{2}{5}, \qquad \text{TT reservoir fraction} = \frac{2}{5}.} \tag{31}$$

The large-N limit sharpens further.

Corollary 6 (Beta visibility laws)**.** *Define the scalar and divergence visibility fractions*

$$\eta_{\mathrm{slip}} := \frac{\xi_S^2}{\xi_S^2 + \xi_{V_1}^2 + \xi_{V_2}^2 + \xi_+^2 + \xi_\times^2}, \tag{32}$$

$$\eta_{\mathrm{div}} := \frac{\xi_S^2 + \xi_{V_1}^2 + \xi_{V_2}^2}{\xi_S^2 + \xi_{V_1}^2 + \xi_{V_2}^2 + \xi_+^2 + \xi_\times^2}. \tag{33}$$

In the large-N Gaussian limit on STF space,

$$\eta_{\mathrm{slip}} \xrightarrow{d} \mathrm{Beta}\left(\frac{1}{2}, 2\right), \qquad \eta_{\mathrm{div}} \xrightarrow{d} \mathrm{Beta}\left(\frac{3}{2}, 1\right). \tag{34}$$

In particular,

$$\mathbb{E}[\eta_{\mathrm{slip}}] = \frac{1}{5}, \qquad \mathbb{E}[\eta_{\mathrm{div}}] = \frac{3}{5}. \tag{35}$$

Proof. The five STF coefficients become independent centred Gaussians with a common variance. Ratios of chi-square blocks yield Beta laws. □

A typical mode is therefore more slip-silent than the mean alone suggests.

6 Quadrupolar stochastic slip floor

The current slip source is

$$q(x) := D^{ij}\Pi^{(J)}_{ij}(x) = D^{ij}\big(\rho_J(x)\Delta M_{ij}(x)\big). \tag{36}$$

In Fourier space, define the STF contraction tensor

$$K_{ij}(k) := k_i k_j - \frac{1}{3}|k|^2\delta_{ij}. \tag{37}$$

Then

$$\widehat{q}(k) = K_{ij}(k)\,(\widehat{\rho_J\Delta M_{ij}})(k). \tag{38}$$

Assume the filtered current-builder induces a stationary isotropic correlation kernel C_ℓ so that

$$\mathbb{E}\Big[\widehat{\Delta M}_{ij}(p)\widehat{\Delta M}_{kl}(p')^*\Big] = (2\pi)^3\delta(p-p')\,\frac{2}{15N}\,\widehat{C}_\ell(p)P^{\mathrm{STF}}_{ij,kl}. \tag{39}$$

Theorem 7 (Slip-source cross-spectrum)**.** *Conditioned on the density loading profile ρ_J,*

$$\mathbb{E}\big[\widehat{q}(k)\widehat{q}(k')^* \mid \rho_J\big] = \frac{4}{45N}|k|^2|k'|^2P_2(\hat{k}\cdot\hat{k}')\int \mathrm{d}p\,\widehat{\rho}_J(k-p)\overline{\widehat{\rho}_J(k'-p)}\widehat{C}_\ell(p), \tag{40}$$

where $P_2(\mu) = \frac{1}{2}(3\mu^2-1)$ is the Legendre quadrupole.

Proof. The only nontrivial tensor contraction is

$$K_{ij}(k)K_{kl}(k')P^{\mathrm{STF}}_{ij,kl} = |k|^2|k'|^2\left((\hat{k}\cdot\hat{k}')^2 - \frac{1}{3}\right) = \frac{2}{3}|k|^2|k'|^2P_2(\hat{k}\cdot\hat{k}'). \tag{41}$$

The rest is the convolution induced by multiplication by ρ_J in real space. □

Because

$$\widehat{S}_J(k) = 12\pi G_N|k|^{-4}\widehat{q}(k), \tag{42}$$

the current-induced slip field inherits the same quadrupolar angular kernel.

Corollary 8 (Slip-floor spectrum)**.** *At equal modes,*

$$\mathbb{E}\Big[|\widehat{S}_J(k)|^2 \mid \rho_J\Big] = \frac{64\pi^2G_N^2}{5N}\frac{1}{|k|^4}\left(\widehat{C}_\ell * |\widehat{\rho}_J|^2\right)(k). \tag{43}$$

If

$$\widehat{\Psi}_J(k) = -\frac{4\pi G_N}{|k|^2}\widehat{\rho}_J(k), \tag{44}$$

then the spectral floor ratio is

$$\frac{\mathbb{E}[|\widehat{S}_J(k)|^2 \mid \rho_J]}{|\widehat{\Psi}_J(k)|^2} = \frac{4}{5N}\,W_\ell(k;\rho_J), \qquad W_\ell(k;\rho_J) := \frac{(\widehat{C}_\ell * |\widehat{\rho}_J|^2)(k)}{|\widehat{\rho}_J(k)|^2}. \tag{45}$$

In the resolved isotropic regime, $W_\ell \approx 1$, so the RMS amplitude ratio is approximately

$$\frac{\sqrt{\mathbb{E}[|\widehat{S}_J(k)|^2 \mid \rho_J]}}{|\widehat{\Psi}_J(k)|} \approx \frac{2}{\sqrt{5N}}. \tag{46}$$

Thus good isotropic coarse-graining does not remove the resolved slip floor. It drives it to a universal N^{-1} spectral floor.

Morphology dependence through spectral curvature

Write

$$P(k) := |\widehat{\rho}_J(k)|^2. \tag{47}$$

If the correlation kernel is normalised and isotropic with second spectral moment σ_p^2, then a standard Taylor expansion gives

$$W_\ell(k;\rho_J) = 1 + \frac{\sigma_p^2}{6}\frac{\Delta P(k)}{P(k)} + O(\sigma_p^4) = 1 + \frac{\sigma_p^2}{6}\Big(\Delta\log P(k) + |\nabla\log P(k)|^2\Big) + O(\sigma_p^4). \tag{48}$$

So N controls the leading floor, while morphology enters through local spectral curvature on the correlation scale set by the filter.

7 Mixed deterministic baseline and stochastic floor

Write the total slip field as

$$S = S_s + S_J, \tag{49}$$

where S_s is the deterministic scalar baseline and S_J is the centred current floor. In the static loading regime,

$$\Psi = \Psi_s + \Psi_J = (1+f)\Psi_s. \tag{50}$$

Define the baseline ratio

$$r_0(x) := \frac{|S_s(x)|}{|\Psi_s(x)|} \tag{51}$$

and the local floor amplitude

$$a_f(x) := \frac{2f}{\sqrt{5N}}\sqrt{W_\ell(x)}. \tag{52}$$

In the large-N regime, the hidden STF coefficients become Gaussian, so a local normal form is

$$R_f(x) \approx \frac{1}{1+f}\left|r_0(x) + a_f(x)Z(x)\right|, \tag{53}$$

with Z approximately standard normal.

Proposition 9 (Quantile crossover law)**.** *Assume $a_f(x) \ll r_0(x)$. Then for any fixed quantile $p \in (0,1)$,*

$$Q_p(R_f(x)) = \frac{1}{1+f}\Big(r_0(x) + a_f(x)z_p\Big) + O\left(\frac{a_f(x)^2}{(1+f)r_0(x)}\right), \tag{54}$$

where $z_p = \Phi^{-1}(p)$. In particular,

$$Q_{50}(R_f) = \frac{r_0}{1+f} + O\left(\frac{a_f^2}{(1+f)r_0}\right), \tag{55}$$

and

$$Q_{90}(R_f) = \frac{r_0 + 1.28155\,a_f}{1+f} + O\left(\frac{a_f^2}{(1+f)r_0}\right). \tag{56}$$

If instead $a_f \gg r_0$, then

$$Q_p(R_f) \to \frac{2}{\sqrt{5N}}\sqrt{W_\ell}\,q_p^{|Z|}, \qquad q_p^{|Z|} = \Phi^{-1}\left(\frac{1+p}{2}\right). \tag{57}$$

Hence the crossover occurs near

$$f_*(x) \sim \frac{r_0(x)\sqrt{5N}}{2\sqrt{W_\ell(x)}}. \tag{58}$$

Proof. Expand the absolute-value normal form in the small-noise regime and use the folded-normal asymptotics in the large-noise regime. □

This gives a concrete prediction: the first visible breakdown of the clean $(1+f)^{-1}$ law should appear in upper masked quantiles before it appears in the median.

8 Coarse-graining stability

The hidden-visible decomposition is exact under post-formed isotropic smoothing. If C_ℓ acts on an already formed STF field by a radial Fourier multiplier $m_\ell(|k|)$, then every STF coefficient at each mode is multiplied by the same scalar. Hence the scalar-visible line, the divergence-visible three-plane, and the TT reservoir are preserved exactly.

The actual builder is nonlinear because it filters a vector field first and normalises afterwards. Write

$$u(v) := \frac{v}{|v|}, \qquad A(v) := u(v)u(v)^\top - \frac{1}{3}I. \tag{59}$$

For a centred isotropic filter on smooth fields,

$$C_\ell v = v + \frac{\ell^2}{2}\Delta v + O(\ell^4). \tag{60}$$

Assuming $|v| \geq c > 0$, one finds

$$A(C_\ell v) - C_\ell A(v) = O(\ell^2) \tag{61}$$

as $\ell \to 0$.

Proposition 10 (Builder commutator scaling)**.** *Under smoothness and nondegeneracy assumptions on the pre-normalised filtered vector field, the filter-normalise-build pipeline preserves the hidden-visible STF splitting up to an $O(\ell^2)$ commutator.*

This is the gravity-side analogue of the coarse-graining commutator scaling already present in the Converse Madelung framework. It supports the claim that the slip-silent sector is renormalisation-stable under good isotropic coarse-graining.

9 Physics interpretation

The results can be compressed into three physical statements.

First, a hidden current sector can load the Newtonian channel strongly while remaining mostly invisible to scalar slip because scalar slip sees only one STF direction per mode.

Second, low scalar visibility is statistically generic in the isotropic large-N regime. It is not a tuned corner of hidden-stress space.

Third, the current-induced slip field is not arbitrary noise. It carries a universal quadrupolar angular kernel and an N^{-1} floor that survives good coarse-graining on resolved modes.

Together these results suggest that the correct ontology is not "one hidden completion" but

$$\text{rigid Fisher backbone} + \text{hidden completion class/observation kernel.} \tag{62}$$

10 What is proved here and what remains to be frozen

The modewise observation filtration, the STF covariance projector law, the visibility fractions, the Beta visibility laws, the quadrupolar slip-floor spectrum, and the mixed baseline-plus-floor crossover law are the hard technical core of the note.

The full abstract quotient theorem for arbitrary UIH coarse-grainings, and the strongest version of the builder commutator statement, remain one step away from being frozen as full-scale general theorems. They are therefore kept here in the most conservative form compatible with the derivations already completed.

11 References

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